Directed Search and the Baily-Chetty Formula

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1 Directed Search

Random search: firms and workers bump into each other at random.

Directed search: workers can direct search to the sorts of jobs they are interested: one market for plumbers, another for economists

Random search: once firm and worker meet, some sort of bargaining determines what happens, how wages are set.

Directed search: firms promise wages to workers (or expected values in a dynamic setting), assumption that firms have commitment

Directed search: even if firms and workers are all equally productive, can imagine different labor markets with firms promising different wages

firms offering higher wages get more applicants, and fill slots more quickly

workers searching in higher wage markets can expect to wait longer

2 Simple static model

Continuum of measure one of identical workers, identical potential firms

Firms and workers start out unmatched

Worker search is costless

Firms can pay ϕ to post a vacancy

If they hire a worker, worker produces z

Potential labor markets indexed by wage w

Competition => firms in active labor market make zero expected profits:

$$\phi = q(z - w)$$

Suppose workers risk averse. Govt runs UI scheme with benefits κz financed by lump-sum tax τ

$c^w = w - \tau$

$c^u = \kappa z$

Suppose there are u workers searching in a particular market and v vacancies

Assume number of matches is

$$\max\{A\sqrt{uv}, u\}$$

(number of matches cannot exceed u)

So

$$q = A\sqrt{\frac{u}{v}}$$

and the probability a worker finds a match is

$$\begin{aligned} p &= A\sqrt{\frac{v}{u}} \\ &= \frac{A^2}{p} \end{aligned}$$

Worker solves

$$\max_p \{pu(w-\tau) + (1-p)u(\kappa z)\}$$

s.t.

$$\phi = \frac{A^2}{p}(z-w)$$

(idea is that workers can only search for jobs in market into which firms can break even posting vacancies)

FOC

$$u(w-\tau) - u(\kappa z) + pu'(w-\tau)\frac{\partial w}{\partial p} = 0$$

$$\begin{aligned} w &= z - \frac{p\phi}{A^2} \\ \frac{\partial w}{\partial p} &= -\frac{\phi}{A^2} \end{aligned}$$

Can verify there is a unique p that satisfies the FOC => only one type of labor market active in equilibrium.

Note that if UI is very generous $u(w-\tau) - u(\kappa z)$ will be small, and thus p will also be small.

Naturally, workers take τ and w as given when making their choices.

3 What is the optimal replacement rate?

The government faces a budget constraint

$$p\tau = (1-p)\kappa z$$

$$
\begin{aligned}
\tau &= \frac{\kappa z}{p} - \kappa z \\
\frac{d\tau}{d\kappa} &= \frac{\partial \tau}{\partial \kappa} + \frac{\partial \tau}{\partial p}\frac{\partial p}{\partial \kappa}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \tau}{\partial \kappa} &= \frac{(1-p)z}{p} \\
\frac{\partial \tau}{\partial p} &= -\frac{\kappa z}{p^2}
\end{aligned}
$$

Planner solves

$$\max_{\kappa} \{pu(w-\tau) + (1-p)u(\kappa z)\}$$

s.t.

$$p\tau = (1-p)\kappa z$$

internalizing that $p = p(\kappa)$, $\tau = \tau(p(\kappa), \kappa)$, $w = w(p(\kappa))$.

FOC is

$$[u(w-\tau) - u(\kappa z)]\frac{\partial p}{\partial \kappa} + pu'(w-\tau)\left(\frac{\partial w}{\partial p}\frac{\partial p}{\partial \kappa} - \left(\frac{\partial \tau}{\partial \kappa} + \frac{\partial \tau}{\partial p}\frac{\partial p}{\partial \kappa}\right)\right) + (1-p)u'(\kappa z)z = 0$$

This looks complicated. But note that the worker's FOC means that some of these terms cancel out

Dropping those terms, we are left with

$$-pu'(w-\tau)\left(\frac{\partial \tau}{\partial \kappa} + \frac{\partial \tau}{\partial p}\frac{\partial p}{\partial \kappa}\right) + (1-p)u'(\kappa z)z = 0$$

Substitute in the expressions for $\frac{\partial \tau}{\partial \kappa}$ and $\frac{\partial \tau}{\partial p}$:

$$-pu'(w-\tau)\left(\frac{(1-p)z}{p} - \frac{\kappa z}{p^2}\frac{\partial p}{\partial \kappa}\right) + (1-p)u'(\kappa z)z = 0$$

Note that

$$u = 1-p$$

$$\frac{\partial \tau}{\partial p} = -\frac{\kappa z}{p^2} = \frac{-\kappa z}{(1-u)^2}$$

$$
\begin{aligned}
u \times (u'(\kappa z) - u'(w-\tau)) \times z &= -u'(w-\tau)\frac{\kappa z}{(1-u)}\frac{\partial p}{\partial \kappa} \\
&= u'(w-\tau)(1-u)\frac{\partial \tau}{\partial p}\frac{\partial p}{\partial \kappa}
\end{aligned}
$$

This is very intuitive. At the optimum, consider the costs and benefits of increasing κ marginally.

Absent any change in p, consumption for u unemployed workers would go up by z, which we could think about as being financed by an equal size reduction in consumption for workers. The welfare gain from this is the LHS of the above expression.

But if the planner raises κ, there is an equilibrium change in p (it goes down) which necessitates an additional increase in the tax τ, which further depresses consumption of $(1-u)$ workers by the amount $\frac{\partial \tau}{\partial p}\frac{\partial p}{\partial \kappa}$.

Let

$$\begin{aligned} \varepsilon_{u,\kappa} &= \frac{\partial u}{\partial \kappa}\frac{\kappa}{u} \\ &= -\frac{\partial p}{\partial \kappa}\frac{\kappa}{u} \end{aligned}$$

denote the elasticity of the unemployment wrt κ. We can write the BC formula as

$$(u'(\kappa z) - u'(w-\tau)) = u'(w-\tau)\frac{\varepsilon_{u,\kappa}}{(1-u)}$$

The terms in this equation are estimable, so we can see whether current benefits are too high or too low.

There is a big empirical literature that tries to estimate the elasticity of the unemployment rate to the replacement rate.

The difference in marginal utilities depends on two things: (i) the assumed curvature in the utility function, and (ii) the gap between $w-\tau$ and κz.

There are two ways to try to measure that gap. One is to estimate the actual replacement rate (perhaps 40%). The other is to look at how much worker's consumption declines when they experience a spell of unemployment (perhaps 10%).