Deficit Spending in an OLG Economy

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1 An OG economy

How does the analysis change in an OG economy?

Consider a two period OG setup (each period is say 30 years)

Suppose a tree drops one unit of fruit each period

Fraction $1-\theta$ is labor income for the young

Fraction θ accrues to the owner of the tree, which has a price P_t

Absent the government, budget constraints are

$$\begin{aligned} C_t^Y &= (1-\theta) - P_t S_{t+1} \\ C_t^o &= (P_t + \theta) S_t \end{aligned}$$

where S_{t+1} is the number of shares the young purchase (which must be one in equilibrium)

Household preferences are

$$\log C_t^Y + \beta \log C_{t+1}^o$$

Allocations must satisfy

$$\begin{aligned} \frac{1}{C_t^Y} &= \beta R_{t+1} \frac{1}{C_{t+1}^o} \\ R_{t+1} &= \frac{\theta + P_{t+1}}{P_t} \end{aligned}$$

$$C_t^Y + \frac{C_{t+1}^o}{R_{t+1}} = (1-\theta)$$

Thus, in steady state, absent government intervention, we have

$$C_t^Y (1+\beta) = (1-\theta)$$

which implies

$$\begin{aligned} P_t &= (1-\theta) - \frac{(1-\theta)}{1+\beta} \\ &= \frac{\beta(1-\theta)}{1+\beta} \end{aligned}$$

and

$$
\begin{aligned}
R &= \frac{\theta(1+\beta)}{(1-\theta)\beta}+1 \\
&= \frac{\theta+\beta}{(1-\theta)\beta}
\end{aligned}
$$

Note that reducing β pushes up R. But note also that θ plays an important role: a larger θ (larger supply of assets) raises R.

Note that we cannot get negative real rates in this economy. Why?

Now introduce government debt and government spending

Debt and the tree must offer the same return (absent uncertainty)

Suppose the government spends G_0 at date 0 financed by issuing debt (a surprise)

The young are the only possible buyers of this debt

Their budget constraint is

$$
C_0^Y = (1-\theta) - P_0 S_1 - Q_0 B_1
$$

where we know that

$$
G_0 = Q_0 B_1
$$

We also know, that in this particular economy, the young optimally choose the same consumption irrespective of the interest rate (as long as they never face any taxes)

So we have, in equilibrium

$$
C_0^Y = \frac{(1-\theta)}{(1+\beta)} = (1-\theta) - P_0 - G_0
$$

which implies

$$
P_0 = \frac{\beta(1-\theta)}{(1+\beta)} - G_0
$$

so deficit public spending at date 0 reduces tree prices one-for-one. What is the intuition? Only the young buy assets, and their total demand for saving is fixed. So more asset supply (i.e., more debt issued) means lower asset prices.

We also know that

$$
\begin{aligned}
C_0^o &= \theta + P_0 \\
&= \theta + \frac{\beta(1-\theta)}{(1+\beta)} - G_0
\end{aligned}
$$

which tells us that the old take the entire hit from extra government purchases at date 0.

Let's assume that at date 1 the debt is paid off with a lump-sum tax on the young

$$
T_1 = B_1
$$

For the young, we get the usual decision to consume a fraction of permanent income, which is now reduced by T_1 :

$$C_1^Y = \frac{1-\theta-T_1}{1+\beta}$$

From their budget constraint, consumption of the old is

$$\begin{aligned} C_1^o &= P_1+\theta+B_1 \\ &= P_1+\theta+T_1 \end{aligned}$$

From the resource constraint

$$C_1^o = 1 - C_1^Y$$

We also have the FOC for the young at date 0

$$\begin{aligned} \frac{1}{C_0^Y} &= \beta\frac{P_1+\theta}{P_0}\frac{1}{C_1^o} \\ &= \beta\frac{1}{Q_0}\frac{1}{C_1^o} \end{aligned}$$

where

$$Q_0 B_1 = Q_0 T_1 = G_0$$

Combining we know that

$$\begin{aligned} \frac{P_0}{P_1+\theta}B_1 &= G_0 \\ B_1 &= G_0\frac{P_1+\theta}{P_0} \end{aligned}$$

Can we pin down P_1 from the FOC for the young at $t=0$?

$$\begin{aligned} \frac{1}{C_0^Y} &= \beta\frac{P_1+\theta}{P_0}\frac{1}{P_1+\theta+G_0\frac{P_1+\theta}{P_0}} \\ \frac{1}{\frac{(1-\theta)}{(1+\beta)}} &= \beta R_1\frac{1}{P_0R_1+G_0R_1} \\ \frac{1}{\frac{(1-\theta)}{(1+\beta)}} &= \beta R_1\frac{1}{P_0R_1+G_0R_1} \\ \frac{(1+\beta)}{(1-\theta)} &= \beta\frac{1}{P_0+G_0} = \beta\frac{1}{\frac{\beta(1-\theta)}{(1+\beta)}} \end{aligned}$$

so this equation is automatically satisfied for any R_1 (equivalently, for any P_1). The logic is that a higher R boosts returns and consumption C_1^o by the same amount

So what does pin down P_1?

At date 2 (and thereafter) there is no debt and no taxes. So from optimality for the young and the resource constraint, we have

$$\begin{aligned} C_2^Y &= \frac{1-\theta}{1+\beta} \\ C_2^o &= \frac{\beta+\theta}{1+\beta} \end{aligned}$$

And we also know that

$$C_2^Y + P_2 = (1-\theta)$$

which implies

$$\begin{aligned} P_2 &= (1-\theta) - \frac{1-\theta}{1+\beta} \\ &= \frac{\beta(1-\theta)}{1+\beta} \end{aligned}$$

which is the steady state price

So the price sequence P_1 must satisfy the FOC for the young at $t = 1$

$$\begin{aligned} \frac{1}{\frac{1-\theta-G_0\frac{P_1+\theta}{P_0}}{1+\beta}} &= \beta\frac{P_2+\theta}{P_1}\frac{1}{\frac{\beta+\theta}{1+\beta}} \\ \frac{1}{1-\theta-G_0\frac{P_1+\theta}{\frac{\beta(1-\theta)}{(1+\beta)}-G_0}} &= \beta\frac{\frac{\beta(1-\theta)}{1+\beta}+\theta}{P_1}\frac{1}{\beta+\theta} \\ \frac{\frac{\beta(1-\theta)}{(1+\beta)}-G_0}{(1-\theta)\left(\frac{\beta(1-\theta)}{(1+\beta)}-G_0\right)-G_0(P_1+\theta)} &= \beta\frac{\frac{\beta(1-\theta)}{1+\beta}+\theta}{P_1}\frac{1}{\beta+\theta} \end{aligned}$$

which can be solved for P_1

Consider an example. Suppose $\theta = 0.2$ $\beta = 0.6$ (which implies that in steady state $C^Y = C^o$)

In steady state, we have $R = \frac{\theta+\beta}{(1-\theta)\beta} = 1\frac{2}{3}$ (so $\beta R = \frac{5}{3}\frac{3}{5} = 1$) $R = \frac{5}{3}$

Suppose $G_0 = 0.1$

That implies

$P_0 = 0.2$

$P_1 = 0.221$

$P_2 = 0.3$

$P_3 = 0.3$

...

This implies

$$\begin{aligned}
C_0^Y &= \frac{1-\theta}{1+\beta} = 0.5 \\
C_0^o &= 1 - C_0^o - G_0 = 0.4
\end{aligned}$$

$$\begin{aligned}
C_1^Y &= \frac{1-\theta-G_0\frac{P_1+\theta}{P_0}}{1+\beta} = 0.368 \\
C_1^o &= P_1 + \theta + G_0\frac{P_1+\theta}{P_0} = 0.632
\end{aligned}$$

$$\begin{aligned}
C_2^Y &= 0.5 \\
C_2^o &= 0.5
\end{aligned}$$

So this particular scheme:
(1) hurts the old at $t = 0$
(2) hurts the newborn at $t = 1$ (they pay more taxes)
(3) benefits the newborn at $t = 0$ (who consume more at $t = 1$)
Interest rates are high for two periods:
high at date 0 because the young have to buy the debt
high at date 1 because the young at $t = 1$ have to pay off the old debt

$$\begin{aligned}
R_1 &= \frac{P_1+\theta}{P_0} = 2.105 \\
R_2 &= \frac{P_2+\theta}{P_1} = 2.262 \\
R_3 &= \frac{P_3+\theta}{P_2} = 1.667
\end{aligned}$$

2 Homework (optional):

Work things out for the case in which the increase in spending is financed by a tax on the old at $t = 1$?

Answer:
The budget constraints for the young at date 0 are

$$\begin{aligned}
C_0^Y &= (1-\theta) - P_0 S_1 - Q_0 B_1 \\
C_1^o &= S_1(\theta + P_1) + B_1 - T_1
\end{aligned}$$

or

$$\begin{aligned}
C_0^Y &= (1-\theta) - Sav_1 \\
C_1^o &= R_1 Sav_1 - T_1
\end{aligned}$$

Combining the two gives

$$C_0^Y + \frac{C_1^o}{R_1} = (1-\theta) - \frac{T_1}{R_1}$$

The FOC, as usual, is

$$\frac{C_1^o}{C_0^Y} = \beta R_1$$

which implies

$$C_0^Y = \frac{(1-\theta) - \frac{T_1}{R_1}}{1+\beta}$$

We know that

$$\begin{aligned} Q_0 B_1 &= G_0 \\ B_1 &= R_1 G_0 = T_1 \end{aligned}$$

Thus

$$C_0^Y = \frac{(1-\theta) - G_0}{1+\beta}$$

which implies

$$\begin{aligned} C_0^o &= 1 - G_0 - C_0^Y \\ &= \frac{\beta+\theta}{1+\beta} - \frac{\beta G_0}{1+\beta} \end{aligned}$$

So now the consumption of both young and old takes a hit in period 0.

In period 1 the newborn young never face any taxes. So their consumption must be

$$C_1^Y = \frac{1-\theta}{1+\beta}$$

which implies

$$C_1^o = \frac{\beta+\theta}{1+\beta}$$

so we can now compute R_1 using

$$\begin{aligned} R_1 &= \frac{1}{Q_0} = \frac{\theta + P_1}{P_0} = \frac{1}{\beta}\frac{C_1^o}{C_0^Y} \\ &= \frac{1}{\beta}\frac{\frac{\beta+\theta}{1+\beta}}{\frac{(1-\theta)-G_0}{1+\beta}} \\ &= \frac{1}{\beta}\frac{\beta+\theta}{(1-\theta)-G_0} \end{aligned}$$

Returning to the budget constraint for the young at $t = 0$ we have

$$\begin{aligned} C_0^Y &= (1-\theta) - P_0 - Q_0 B_1 \\ \frac{(1-\theta) - G_0}{1+\beta} &= (1-\theta) - P_0 - G_0 \end{aligned}$$

which implies

$$P_0 = \frac{\beta(1-\theta)}{1+\beta} - \frac{\beta G_0}{1+\beta}$$

and thus

$$\frac{\theta + P_1}{P_0} = \frac{1}{\beta} \frac{\beta + \theta}{(1-\theta) - G_0} = \frac{\theta + P_1}{\frac{\beta(1-\theta)}{1+\beta} - \frac{\beta G_0}{1+\beta}}$$

which implies

$$P_1 = \frac{\beta(1-\theta)}{1+\beta}$$

which is the steady state price.

So from period 1 onwards the economy is effectively back in steady state.

So with this financing scheme, extra spending at date 0 reduces utility for the old at date zero, and for the young generation entering at date 0, but does not affect any other generations.