

Public Education and Income Distribution: A Dynamic Quantitative Evaluation of Education-Finance Reform

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Many states are implementing school-finance reforms which will have complex effects on income distribution, intergenerational income mobility, and welfare. This paper analyzes the static and dynamic effects of such reforms by constructing a dynamic general equilibrium model of public-education provision and calibrating it using U.S. data. We examine the consequences of a reform of a locally financed system to a state-financed system which equalizes expenditures per student across districts. We find that this policy increases both average income and the share of income spent on education. Steady-state welfare increases by 3.2 percent of steady-state income. (JEL I22, H42)

Over the last two decades a series of state supreme court rulings and concern over public education have led many states to enact reforms with the aim of reducing inequality of access to quality public education. This process began with the *Serrano* ruling in California in 1971 and to date has overturned the school-financing systems in 16 states. The effect of this litigation, both actual and its threat, has been to increase the role of the state and decrease that of local provision.

These reforms have motivated several researchers to build models aimed at uncovering the effects of changes in finance systems. A noteworthy early example is Robert P. Inman (1978), who carried out a normative analysis of education-financing systems according to several welfare criterion for a model estimated from data for the New York City metropolitan area. His theoretical framework is a static multicommunity model of the sort pioneered by Frank H. Westhoff (1977). This framework has been used more recently to investigate analytically questions in which the focus is on

the incentives for heterogeneous individuals to sort themselves across communities, the determination of the distribution of locally provided goods across localities, and the static efficiency properties of the resulting equilibrium. Papers in this line include Dennis Epple et al. (1984), Timothy J. Goodspeed (1989), Charles A. M. de Bartolome (1990), Epple and Thomas Romer (1991), Roland Benabou (1993), Fernández and Rogerson (1996, 1997a), and Epple and Richard E. Romano (1998).

While we think static considerations are undoubtedly an important component in evaluating finance reforms, in this paper we argue that dynamic considerations may also be significant. The theoretical basis for this claim is simple and has been articulated in many papers (though not necessarily in a multicommunity framework), starting with the seminal works of Gary S. Becker and Nigel Tomes (1979) and Glenn C. Loury (1981), and more recently in Suzanne J. Cooper (1992), Gerhard Glomm and B. Ravikumar (1992), Michele Boldrin (1993), Benabou (1996), and Steven N. Durlauf (1996). The basic argument is easily summarized. Education expenditures on children are a form of human capital investment that yields a return in the form of higher productivity later in life. If there is little opportunity for borrowing against these future earnings to finance current expenditures, inefficiently low investment among children from poor families may result. This

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may be especially true for primary and secondary education in the United States, where the financing of public schools has a large local component and thus is a function of the level of community income. Finance systems which redistribute resources away from the rich and toward the poor may therefore have important implications for the evolution of the income distribution and overall efficiency.

While the theoretical argument above and variants of it seem well understood, there has been little work attempting to assess its quantitative significance. The objective of this paper is both to develop a tractable dynamic multicommunity model and to take a first step at evaluating the quantitative impact of education-finance reforms in a dynamic setting.

We model intergenerational dynamics using a two-period-lived overlapping generations model. In each period there are a large number of families, each consisting of one old member (parent) and one young member (child). An old individual's income is determined by the education received when young and an idiosyncratic random shock. Taking their income as given, old individuals decide in which community to reside, and each community then decides via majority vote the amount of resources to devote to public education. This determines the income distribution for the next generation of adults and the process repeats itself every period. We solve for a steady-state equilibrium for this economy. Although tractability does limit the scope of the model we consider, it incorporates four features which we consider central to an analysis of public education finance in the United States. First, there is substantial heterogeneity of income across households. Second, individuals are mobile across communities. Third, public education is provided at the community level and fourth, funding for public education is largely determined at the local level.

The school-finance system described above is one of pure local financing. The steady-state equilibrium under this system has communities stratified by income, i.e., communities correspond to nonoverlapping intervals of income from the income distribution. Spending on education is perfectly correlated with community income, so richer communities have higher quality education than do poorer com-

munities. It follows that children from higher-income families have higher expected income than do children from poorer families.

Under some specifications of the model's mapping from expenditures on education this period to the income distribution next period, and holding total spending on education (exogenously) constant, the economy's total income next period would be greater if this period's expenditures on education were divided equally across all students. The model thus captures the possibility that a centralized system may offer an efficiency gain relative to a local system.¹ It offers little guidance, however, as to whether this outcome is likely, however, since policy reform will presumably bring about changes on many dimensions, including the share of income devoted to education, the composition of communities, and housing prices. Analytically one cannot say much about how the equilibria under the two financing systems compare nor whether the comparison can be expected to yield quantitatively important differences. To explore this further, we calibrate a two-community version of the model described above to U.S. data.

Empirical work provides a substantial amount of information to guide a reasonable selection of parameter values for our model. Our calibration uses information on the (cross-sectional) elasticity of education expenditures per student with respect to community mean income, the elasticity of (subsequent) earnings with respect to quality of education when young, price elasticities of housing demand and supply, mean and median income, and expenditure shares for housing and education.

In our calibrated model we analyze a reform in which local financing of education is replaced with state financing which distributes education resources equally per student across communities. The benchmark specification of our calibrated model yields the following results: relative to the case of pure local financing, we find that a policy of state financing leads to higher average income in the steady state, higher average spending on education,

¹ It does not guarantee this, however, since among other things this depends on what would happen to total expenditures.

and higher welfare. The magnitude of the welfare improvement measured in terms of steady-state income is 3.2 percent, which is a large gain relative to that found for many policies.² We also trace out what happens along the transition path to the new steady-state equilibrium and examine its welfare implications. We find that for annual discount rates smaller than 8.2 percent, a social planner would choose to implement the policy change. We note, however, that our calibrated model does not predict that a referendum held among the current old generation would lead to adoption of the state-financing system. On the contrary, such a motion would be rejected by the old voters!

Section I describes the benchmark model. Section II discusses the calibration of this model. Section III reports the results of the calibration and of the policy reform carried out in the calibrated model. Section IV performs a sensitivity analysis, and Section V concludes.

I. The Model

The economy is populated by a sequence of two-period-lived overlapping generations. A continuum of agents with total mass equal to one is born in every time period. Each individual belongs to a household consisting of one old person (the parent) and one young person (the child). All decisions are made by old individuals, each of whom has identical preferences given by:

$$(1) \quad u(c, h) + Ew(y_c),$$

where c is consumption of a private good, h is consumption of housing services, E is an expectations operator, and y_c is next period's income of the household's young individual. The function u is assumed to be strictly concave, increasing in each argument, twice continuously differentiable, and defines preferences over c and h that are homothetic.³ The

function w is twice continuously differentiable, increasing and concave.

Individual income is assumed to take one of I values: y_1, y_2, \dots, y_I , with $y_1 < y_2 < \dots < y_I$. An individual's income when old is determined by q —the quality of education obtained when young—and an idiosyncratic shock.⁴ The probability that an individual has income y_i when old, given an education of quality q when young, is equal to $\phi_i(q)$. Define $v(q)$ by:

$$(2) \quad v(q) = Ew(y_c) = \sum \phi_i(q)w(y_i).$$

Preferences can then be defined over c, h , and q :

$$(3) \quad u(c, h) + v(q).$$

We assume that v is increasing, concave, and twice continuously differentiable.

Old individuals must choose a community in which to reside. There are two communities.⁵ These are indexed by j , and referred to as C_j , $j = 1, 2$. Each community j is characterized by a proportional tax t_j on housing expenditures, a quality of education q_j , and a net-of-tax housing price p_j . Proceeds from the tax are used exclusively to finance local public education. We assume that the quality of public education is equal to per-pupil spending on education. All residents of a given community receive the same quality of education; education cannot be privately supplemented.⁶

⁴ Thus, we abstract away from any possible peer effects (i.e., the possibility that who you go to school with matters) and parental characteristics other than income. Quantitative evidence on peer effects is mixed. de Bartolome (1990) summarizes empirical findings and provides a theoretical analysis of peer effects in a multicomunity model as does Benabou (1993). The role of parental characteristics has been studied extensively, and many studies find them to be significant. We abstract from them here, however, to focus our analysis and simplify the model.

⁵ Our theoretical analysis applies equally well with any number of communities. We restrict attention to two communities because of computational considerations relevant for the subsequent quantitative analysis.

⁶ In general it is difficult to incorporate private education in a model with majority vote, as a voting equilibrium may not exist without additional assumptions. See Epplé and Romano (1996, 1998) and Glomm and Ravikumar (1998) for models that incorporate a private-education option.

² For a state with a hybrid of the two extreme finance systems, this calculation may exaggerate the potential gains associated with moving to a centralized system.

³ Homothetic preferences are assumed since they simplify computation; they are in no way essential.

Each community has its own housing market, with supply of housing in C_j given by $H_j^s(p_j)$. Note that this function is allowed to differ across communities, reflecting differences in land endowments and other factors. We assume that H_j^s is increasing, continuous, and equal to zero when p_j is zero. The gross-of-tax housing price in C_j is given by $\pi_j = (1 + t_j)p_j$. How to treat housing is a potentially difficult issue. As shown in Epplé and Romer (1991), the outcomes of policies can depend on whether agents are owners or renters. While we suspect that this issue is likely to be less critical for the type of policies we are interested in examining, our main problem with incorporating owner-occupants is that it adds an additional state variable to the model. Consequently we assume that agents rent housing and that the latter depreciates completely at the end of the renter's lifetime. Furthermore, while the welfare of the owners of housing services is taken into account in our policy analysis, so as not to introduce further complications these agents are assumed to live outside the two communities and simply consume their rental income.

In each period the interaction among individuals and communities can be described as a three-stage game of the following form. In the first stage, all (old) individuals simultaneously choose a community ($C_j, j = 1, 2$) in which to reside. In the second stage, individual residence decisions are fixed and communities choose tax rates through a process of majority vote.⁷ In the third stage, individuals make their housing and consumption choices and young individuals receive education in the community in which their parent has chosen to reside. At the end of the period, uncertainty about next period's income is resolved (note that this occurs before the still-young individuals make their own residence decision of the following period). Then time rolls forward and the three-stage game is repeated with the previous period's young individuals becoming this period's old individuals.⁸

⁷ This assumption implies that each individual takes the composition of the community as given when voting. This greatly simplifies the strategic interactions between communities.

⁸ While the timing of choices in this model—tax rate first followed by housing and consumption—may appear

The utility function chosen in (1) makes this model tractable since it allows us to solve this infinite horizon game period by period. For any given period and income distribution, we can solve the three-stage game by backward induction. Note that from an individual's perspective, a community is completely characterized by the pair (π, q) . Thus, an individual with income y given (π, q) has an indirect utility function $V(\pi, q; y)$ defined by:

$$(4) \quad V(\pi, q; y) = \text{Max}_{c, h} u(c, h) + v(q)$$

$$\text{s.t. } \pi h + c \leq y, c \geq 0, h \geq 0,$$

where c has been chosen as numeraire.⁹ For any equilibrium outcome $(\pi_1^*, q_1^*, \pi_2^*, q_2^*)$ each individual must reside in the community that yields her the greater utility.

Define $h(\pi, y)$ to be the individual housing demand resulting from the maximization problem in (4). By homotheticity h can be written as $g(\pi)y$.¹⁰ Given a set of residents of mass N_j and a tax rate t_j in C_j , the variables q_j and p_j must satisfy:

$$(5a) \quad N_j g(\pi_j) \mu_j = H_j^s(p_j);$$

$$(5b) \quad t_j p_j g(\pi_j) \mu_j = q_j.$$

where μ_j is mean income in C_j . The first equation requires that the housing market clear. The second states that the quality of education q_j equals the per (old) person tax revenue of the community. It is straightforward to show that for any positive value of t_j equation (5a) has a unique solution for p_j . Furthermore, p_j is decreasing in t_j and π_j is increasing in t_j .

unnatural, it allows us to avoid issues of multiple equilibria that could occur if the opposite timing were adopted. See Epplé et al. (1984), however, for conditions that guarantee the existence of a unique equilibrium when the opposite timing assumption is adopted.

⁹ Note that we have implicitly assumed that education is the only technology available by which a parent can contribute to her child's income. Note also that the formulation in (4) makes explicit that there are no capital markets that permit borrowing nor insurance markets for the uncertainty in children's future income.

¹⁰ In what follows we assume that the optimization problem in (4) results in interior solutions for c and h .

The following assumption on preferences greatly facilitates characterization of equilibrium in the three-stage game.¹¹

ASSUMPTION 1: For all (π, y) , $v'(q)/[u_c(y - \pi h(\pi, y), h(\pi, y))h(\pi, y)]$ is increasing in y .

Since $v'(u_c h)$ is the slope of an individual's indifference curve in (q, π) space, Assumption 1 guarantees that this slope is increasing in initial income, i.e., that

$$(6) \quad S = u_{cc}h(1 - \pi h_y) + u_{ch}h_y + u_c h_y < 0.$$

The power of this assumption to characterize equilibrium is seen in the next two propositions which are common in the multi-community literature.

PROPOSITION 1: *Given a set of residents, a majority voting equilibrium over tax rates in a community exists and results in the preferred tax rate of the resident with the median income.*

PROOF:

Since a tax rate implies a gross housing price through the equilibrium condition for the housing market, preferences over (q, t) map into preferences over (q, π) . The results then follow from the property of indifference curves discussed previously. See Epple and Romer (1991) and Fernández and Rogerson (1996) for detailed proofs in slightly different contexts.

PROPOSITION 2: *If in equilibrium q_1^* is not equal to q_2^* and neither community is empty, then: (i) $(\pi_1^*, q_1^*) \gg (\pi_2^*, q_2^*)$; (ii) all individuals in C_1 have income at least as great*

as any individual in C_2 , where C_1 is defined as the community with the higher value of q .

PROOF:

(i) If $\pi_1^* < \pi_2^*$ and $q_1^* \geq q_2^*$, then all individuals prefer to live in C_1 , which contradicts the assumption that no community is empty. (ii) This follows directly from Assumption 1 regarding the slope of indifference curves in (q, π) space as a function of y .

Proposition 2 implies that an equilibrium with $q_1^* \neq q_2^*$ will be characterized by the co-existence of a community with high-income residents, high gross-of-tax housing prices, and high-quality education; and another community with lower-income residents, lower gross-of-tax housing prices, and a lower quality of education.

Any equilibrium that displays property (ii) of Proposition 2 is said to be a *stratified* equilibrium and is common to multicommunity models.¹² Problems of existence and uniqueness of a stratified equilibrium are endemic to multicommunity models (see, for example, Westhoff [1977, 1979] and Epple et al. [1984] for a discussion). In all of the simulations reported later in the paper, however, the specifications are such that a unique stratified equilibrium exists.

Lastly, we turn to a characterization of the tax rates generated by majority voting. Using (5a) and (5b) one can write $q_i(t, \mu, N)$ and $\pi_i(t, \mu, N)$ as the quality of education and tax-inclusive housing price, respectively, in C_i given a tax rate t , community mean income μ , and a community population of N . The pre-

¹¹ This assumption is a single-crossing condition. While its particular algebraic expression depends on the specific model, it is used by the multicommunity literature to induce separation of individuals and thus allow equilibrium to be characterized. [See, for example, Westhoff (1977) and Fernández and Rogerson (1996, 1997a); see Fernández (1997) for an analysis of the strange comparative static properties of these systems.]

¹² There may also exist equilibria which are not stratified. For example, given identical housing-supply functions there always exists an equilibrium in which the two communities are identical, i.e., half of each income group resides in each community, resulting in equal tax rates, prices, and quality of education. In the analysis that follows, however, we only consider stratified equilibria. See Westhoff (1979) and Fernández and Rogerson (1996) for a discussion in a slightly different context of how requiring stability of equilibria can eliminate all nonstratified equilibria.

ferred tax rate for an individual with income y is determined by:

$$(7) \quad \begin{aligned} &\text{Max}_{t=0} u(y - \pi h, h) + v(q) \\ &\text{s.t. } h = g(\pi)y \\ &H^s(p) = Ng(\pi)\mu \\ &q = tp g(\pi)\mu \\ &\pi = p(1 + t). \end{aligned}$$

Using the envelope theorem, the first-order condition for this problem implies:

$$(8) \quad u_c h \pi_t = v' q_t.$$

In a stratified equilibrium C_1 has both higher mean and higher median income than C_2 . Two comparative statics exercises, therefore, are of interest; how is the tax rate that solves (8), denoted by \tilde{t} , affected by: (i) a change in y ; and (ii) a change in μ ? Straightforward calculation yields:

$$(9) \quad \partial \tilde{t} / \partial y = \pi_t S / D > 0$$

and

$$(10) \quad \frac{\partial \tilde{t}}{\partial \mu} = \frac{1}{D} \left\{ [u_{cc} \pi_\mu (h + \pi h_\pi) - u_{ch} h_\pi \pi_\mu] h \pi_t - h u_{c\pi} \pi_{t\mu} - u_{c\pi} h_\pi \pi_\mu \pi_t + (v'' q_\mu + v' q_{t\mu}) \right\} \geq 0,$$

where D denotes the second derivative of the maximand in (7) with respect to t . By the second-order condition, D is nonpositive at a maximum.

The first expression states that higher-income individuals prefer higher tax rates and necessarily, therefore, higher quality education. This follows from our single-crossing assumption on preferences which implies that the willingness to pay higher after-tax prices on housing in order

to obtain a given increase in the quality of education is increasing in income.

The second expression indicates that an increase in mean income has an ambiguous effect on an individual's preferred tax rate. Thus it is not possible to state whether in equilibrium C_1 must have a higher or lower tax rate than C_2 . This follows from the fact that an increase in μ for a given t and π increases the tax base, providing greater quality. This induces a substitution effect as individuals now wish to increase consumption and housing and decrease q on the margin, but also creates an income effect as the tax-base increase means that a marginal increase in the tax rate increases q by more than before. As will be seen in the next section, evidence on the relationship between community mean income and spending on education suggests that the sign of $\partial t / \partial \mu$ is negative.

Thus far we have discussed the properties of equilibrium of the three-stage game for any period t without making reference to future periods. It is possible to do so since the outcome in period t is independent of the future evolution of the state variable.¹³ Since our larger game simply repeats this three-stage game every period, we need only keep track of the state variable of this game—the income distribution of old agents—which we write as $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_t)$ where λ_t is the fraction (or equivalently mass) of old agents with income equal to y_t . In the next section we focus on the properties of a steady state for the system. If λ_t is the income distribution of old individuals at the beginning of period t , then an equilibrium of the three-stage game in period t generates a beginning-of-period income distribution for period $t + 1$, λ_{t+1} . We denote by $\Lambda(\lambda)$ the set of values for λ_{t+1} that correspond to subgame-perfect equilibria of the three-

¹³ This fact, which greatly simplifies the analysis, follows from the assumption that an old individual cares about the young individual's income rather than utility, thus severing the link between one time period and the next. This is a commonly used device to render this type of analysis tractable. See, for example, Cooper (1992); Glomm and Ravikumar (1992); Durlauf (1996). See Per Krusell and José-Victor Ríos-Rull (1996) for an illustration of the difficulties in relaxing this assumption.

stage game given $\lambda_i = \lambda$. A steady state is a value λ^* such that $\lambda^* \in \Lambda(\lambda^*)$.

It is difficult to say much about the dynamic properties of this model without imposing more structure. We next turn to this task and choose functional forms and parameter values so that we can discuss quantitative as well as qualitative features associated with a change in education finance policy.¹⁴

II. Calibration

The objective of this section is to quantify the effects of a major change in the system of financing education in the context of the model outlined in Section I. To do so it is necessary to specify functional forms for the relationships introduced in the previous section and assign parameter values. We first turn though to a brief description of the computation of equilibrium.

A. Computation of Equilibrium

We use numerical methods to solve for the equilibrium of our model. Given a beginning-of-period income distribution, our algorithm finds all stratified equilibria to the three-stage game played in each period. The key fact used in this procedure is that all potential stratified equilibrium can be parametrized by the fraction of the population that resides in C_1 . We denote this fraction as ρ . Each value of ρ determines the income distributions of the two communities since it partitions the income space into higher-income individuals that reside in C_1 and lower-income individuals that reside in C_2 . Associated with each value of ρ is a highest-income individual in C_2 ; call this value y_{b2} . Let y_{b1} be the lowest income of an individual in C_1 .

Define $W_j(\rho)$ to be the utility of an individual with income y_{bj} residing in C_j given that ρ partitions the residents of the two communities and that each community chooses its tax rate via majority vote. An equilibrium can be depicted as a "crossing" of the two W_j curves.¹⁵ We compute the W_j curves and therefore find all the stratified equilibria. In all our simulations the stratified equilibrium is unique.¹⁶

Given an initial income distribution, repeated application of the above procedure can be used to solve for the entire equilibrium sequence. We look for stable steady-state income distributions by examining the dynamic path for λ that results from each of a large set of initial distributions. In our simulations we find a unique (stable) steady state and convergence always occurs.

B. Functional Forms

Three functional relationships need to be specified: preferences, housing supply, and the effect of the quality of education on subsequent earnings. Our guiding principle in the choice of functional forms is to use, wherever possible, specifications that are commonly used in quantitative analyses. Hence, while we typically have little hard evidence to suggest our functional forms over some alternatives, we think our choices constitute a natural starting point for investigating the quantitative properties of these models. For preferences we assume:

$$(11) \quad u(c, h) = \frac{a_c c^\alpha + (1 - a_c) h^\alpha}{\alpha}$$

$$w(y_c) = \frac{a_q (y_c^\gamma - 1)}{\gamma}$$

$$0 < a_c < 1, \quad \alpha, \gamma \leq 1 \quad a_q > 0.$$

¹⁴ In a related paper Benabou (1996) does obtain some analytical results for a class of models with specific functional form assumptions. For example, he shows that, under some conditions, long-run income is higher under state finance than under local finance. While this result also obtains for our calibrated economy, it is not a property that holds in general for our specification. A key simplifying assumption in Benabou is that all agents have the same preferred tax rate.

¹⁵ Since the y_{bj} are discontinuous functions of ρ (given a discontinuous income distribution), the W_j functions are also discontinuous. Hence the equilibrium need not require $W_1(\rho) = W_2(\rho)$.

¹⁶ It is possible for identical individuals to end up residing in different communities. Although the equilibrium does not uniquely determine what happens to these individuals it does uniquely determine what fraction of them are in each of the two communities.

The specification for $u(c, h)$ is a transformation of a constant-elasticity-of-substitution utility function. Note that Assumption 1 is satisfied if and only if α is strictly negative [given $v'(q) > 0$]. The choice for $w(y_c)$ displays constant relative risk aversion.

We assume identical constant-elasticity housing-supply functions for both communities, i.e.,

$$(12) \quad H_j^s(p_j) = ap_j^b.$$

This specification assumes the same price elasticity for both communities (i.e., b).

The final relationship that needs to be specified is that linking quality of education to subsequent earnings. We assume that each individual's realized income is a draw from a discrete approximation to a lognormal distribution whose mean depends on q . In particular, consider a lognormal distribution of income where log of income has mean $m(q)$ and variance σ^2 and $m(q)$ is defined by:

$$(13) \quad m(q) = y_0 + \frac{B(1+q)^\delta}{\delta} \quad B > 0.$$

Let an ascending vector $[\tilde{y}_1, \dots, \tilde{y}_I]$ partition the distribution of income into $I + 1$ where y_i is contained in $(\tilde{y}_i, \tilde{y}_{i+1})$ for $i = 1, 2, \dots, I - 1$, and $y_I > \tilde{y}_I$. For each community, we transform the continuous distribution in (13) to a discrete distribution over the I income types [hence obtaining the $\phi_i(q)$] by integrating the above lognormal distribution over the interval containing y_i .

A few comments should be noted concerning this choice. First, $B > 0$ implies that expected income is increasing in q . Second, we assume that σ is independent of q .¹⁷ Third, $m(q)$ can be concave or convex in q , depending on whether δ is smaller or larger than one.

C. Parameter Values

We choose parameter values such that the steady-state equilibrium of the model matches

important observations for the U.S. economy. In particular, we require that the steady state match several aggregate expenditure shares, elasticities, and properties of the income distribution for the U.S. economy.

There are three commodities in the model: consumption, housing, and education, and hence two independent expenditure shares. The ratio of annual aggregate housing expenditures to aggregate expenditures on consumption (including housing), H/TC , averaged over 1960–1990 is 0.15, and the average annual ratio of spending on public elementary and secondary education to aggregate expenditures on consumption, E/TC , is 0.053.¹⁸

We match four elasticities: the price elasticities for housing demand and supply ($\varepsilon_{h,\pi}^d$ and $\varepsilon_{h,p}^s$), the elasticity of mean earnings with respect to the quality of education ($\varepsilon_{m,q}$), and the cross-sectional elasticity of community public-education expenditures with respect to community mean income ($\varepsilon_{q,m}$).

John M. Quigley (1979) surveys the literature on urban housing markets. Based on this survey we choose to match a price elasticity of housing demand (gross of taxes) equal to -0.7 and a price elasticity of housing supply equal to 0.5 . Estimates of the demand elasticity range as high as -0.95 , however, and the range of estimates of the supply elasticity is large. Additionally, the mapping between the (implicit) models underlying these elasticity estimates and our model is not exact. Hence we also explore the effect of different price elasticities for our results.¹⁹ Note that the functional form we have chosen for the utility function does not imply a constant demand-price elasticity for housing. By homotheticity, however, the price elasticity of demand for housing is independent of income so that we can use the model's cross-sectional steady-state observations of housing prices and per capita housing quantities to compute the (gross) price

¹⁷ It is simple to allow for the variance to depend on q , but since we are aware of no evidence on this issue we do not include this effect in our empirical work.

¹⁸ Data for housing expenditures and consumption are taken from the *Economic Report of the President*, and data on educational expenditures are taken from various issues of the *Statistical Abstract of the United States*.

¹⁹ Additional empirical studies are surveyed in Edgar O. Olsen (1986).

elasticity.²⁰ We normalize the parameter a in the housing-supply function to equal one.

A key difference between the two communities in our model is that, in equilibrium, C_1 has both higher mean income and quality of education than C_2 . Therefore, from the steady-state equilibrium one can compute a cross-sectional elasticity of (per-student) expenditures on education with respect to community mean income. Many empirical studies obtain estimates of this elasticity (see Inman [1979]; Theodore C. Bergstrom et al. [1982] for surveys). The range of estimates obtained in these studies is 0.24–1.35, with the vast majority of the estimates lying in the narrower range of 0.4–0.8. We choose parameter values so that $\varepsilon_{q,\mu} = 0.62$ when evaluated at the steady-state equilibrium. Note that a value greater than zero but smaller than one is significant since it implies that, *ceteris paribus*, communities with higher mean incomes spend more on education but tax at a lower rate.

To choose a value for the elasticity of mean earnings with respect to education quality we rely on evidence presented by David Card and Alan B. Krueger (1992), George E. Johnson and Frank P. Stafford (1973), and Paul Wachtel (1976). Card and Krueger, using several indicators of quality of schooling across states and time, estimate that a decrease in the student-to-teacher ratio of ten students would increase earnings by 4.2 percent. Over the period 1924–1964 the average annual ratio of teacher's wages to total costs for public elementary and secondary schools was approximately 54 percent and the average annual student-teacher ratio over the same period was 28.0. The resulting estimate of the elasticity of earnings with respect to education expenditures (quality) is approximately 0.18.²¹

²⁰ A demand elasticity less than one in absolute value corresponds to a negative value of α , which is required to satisfy Assumption 1.

²¹ Note that Card and Krueger's elasticity estimates combine two different effects of quality on earnings: an increase in earnings holding years of education constant, and an increase in wages due to increased years of education. While our model abstracts from the effect of quality on years of education, we believe that the combined effect is the appropriate measure.

Wachtel, in a study that examines the returns to schooling using school-district expenditure levels, finds an elasticity of 0.2. Since college expenditures are included as a separate variable in his regressions, it is reasonable to view this estimate as being on the low side to the extent that higher secondary-education expenditures also increase the probability of attending a higher quality college. Johnson and Stafford also find an elasticity estimate of 0.2. In our benchmark calibration we choose $\varepsilon_{m,q} = 0.1911$ and explore the sensitivity of our results to changes in this value in Section IV. We compute the elasticity by using the cross-sectional variation in the steady-state values of q across communities and equation (13) relating q to mean earnings.

The above estimates of the effect of educational resources on future wages are at the upper end of those which have been obtained, and have been subjected to criticism.²² An alternative calibration strategy is to require that the steady-state equilibrium yield a measured rate of return to education that is "reasonable." In a later section we compute this rate of return for both our benchmark specification and for calibrations using alternative values of $\varepsilon_{m,q}$ and show that our benchmark choice of parameter values implies a value for the rate of return to spending on education that is in the lower part of the range that has been provided by empirical work (see Robert J. Willis [1986] for a summary of findings). Generating higher rates of return would require even higher values of $\varepsilon_{m,q}$, so in this sense our benchmark is not based on extreme values.

The final piece of information we use in calibration is data on the income distribution of families from the 1980 Census. We choose the \tilde{y}_i 's to match the commonly used (in thousands) income intervals— $\tilde{y} = (0, 5, 7.5, 10, 15, 20, 25, 35, 50)$ —and set the vector of y_i 's equal to (2.5, 6.75, 8.75, 12.5, 17.5, 22.5, 30, 42.5, 60). Two additional items of information that we match in the steady state of our

²² See in particular Julian R. Betts (1995) and James Heckman et al. (1995). More generally, see James S. Coleman et al. (1966) and Eric A. Hanushek (1986) for discussions on the relationship between resources and educational outcomes.

model are the 1980 Census values of mean and median family income values, equal to 21.4 and 17.9, respectively.²³

Lastly, in our benchmark specification we set $\gamma = 0$, which implies that $v(q)$ can be approximated by the expression:²⁴

$$(14) \quad v(q) \approx a_q \left[y_0 + \frac{B(1+q)^\delta}{\delta} \right].$$

Although this choice of γ is somewhat arbitrary, as we report in Section IV our results are not sensitive to changes in this value.

The items of information described above (two expenditure shares, four elasticities, mean, and median income) and the chosen values of $\gamma = 0$ and $a = 1$ can be used to determine the eight parameter values: a_c , a_q , b , δ , α , B , y_0 , and σ^2 .

There are two qualifications that should be mentioned regarding our calibration procedure. First, although there is a substantial amount of work which provides estimates of the elasticities used in our procedure, the literature typically provides a range of values. Additionally, the structures on which some estimates are based are not always the same as the structure of our model. We deal with these concerns by carrying out a sensitivity analysis.

III. Results

A. Properties of the Benchmark Model

In this section we report the parameter values generated by the calibration described in the previous section and present some additional properties of the steady state and of the dynamics of the system. As noted before, our computations yield a unique equilibrium for the one-period game, a unique stable steady state, and convergence to the steady state.²⁵

²³ When we compute median income in the model we assume that individuals with income y_i are uniformly distributed over the interval $[\bar{y}_i, \bar{y}_{i+1}]$.

²⁴ The fit of this approximation depends on how closely the transformation from a continuous to a discrete distribution preserves the mean of log income.

²⁵ Typically, convergence to the steady state is quite rapid (three or four periods).

TABLE 1—PARAMETER VALUES AND STEADY-STATE VALUES

Parameter values	
Preference parameters:	
$a_c = 0.936, a_q = 0.034, \alpha = -0.6, \gamma = -0.0001$	
Housing-supply parameters:	
$a = 1, b = 0.5$	
Education-earnings relationship:	
$\delta = -3.9, B = 8, y_0 = 3.01, \sigma^2 = 0.63$	
Steady-state values	
$\lambda = (0.02, 0.06, 0.10, 0.21, 0.18, 0.13, 0.15, 0.09, 0.05)$	
mean income = 21.56, median income = 17.91	
$\varepsilon_{h,\pi}^d = -0.6957, \varepsilon_{m,q} = 0.1911, \varepsilon_{q,\mu} = 0.6162$	
$E/TC = 0.0545, H/TC = 0.1448$	

Table 1 reports the parameter values used in the calibration and the steady-state values for several variables and Table 2 provides the steady-state values of the community variables.

Note that, as required for a stratified equilibrium, both quality and the gross price of housing are higher in community one. The net-of-tax price of housing is also higher in C_1 . This is essential to produce stratification since the tax rate in C_1 is lower than that in C_2 as required by $\varepsilon_{q,\mu} < 1$ (given $q_1 > q_2$ and $p_1 > p_2$). In the steady state all individuals with income greater than 22.5 live in C_1 , all individuals with income less than 22.5 live in C_2 , and individuals with income equal to 22.5 are split across the two communities.

Spending per student is nearly twice as large in C_1 as in C_2 . Although there are many metropolitan areas in which this range of expenditures exists, this ratio is somewhat on the extreme side of what is observed in the U.S. data. This is not surprising, however. Our model describes how expenditures would vary across communities if all financing were done at the local level. The fact that differences are not as large in the U.S. data as they are in our calibrated model may simply indicate that state aid does (on average) decrease differences in education expenditures across communities. Furthermore, although the difference

TABLE 2—STEADY-STATE EQUILIBRIUM VALUES

	C_1	C_2
t	0.287	0.512
p	1.468	1.117
π	1.889	1.689
q	1.625	0.881
μ	37.996	14.034
E/TC	0.0447	0.0670
H/TC	0.1557	0.1308
ρ	0.314	
y_b	22.5	

TABLE 3—DISTRIBUTION OF INCOME GENERATED BY COMMUNITY

Income	C_1	C_2
2.5	0.02	0.03
6.75	0.05	0.07
8.75	0.08	0.10
12.5	0.20	0.22
17.5	0.18	0.18
22.5	0.14	0.13
30.0	0.17	0.14
42.5	0.11	0.08
60.0	0.07	0.04
	mean = 23.33	mean = 20.76

in mean income across communities is not out of line with that between central cities and suburbs in several metropolitan areas, our extreme stratification result probably exaggerates the differences in mean incomes across communities in the United States (mean income in C_1 is over twice that in C_2). Given the elasticity of spending on education with respect to community mean income, this results in a large difference in per-capita education expenditures.

Since the quality of education differs across communities, the children of wealthier individuals will belong to a different income distribution when old than the children of poorer individuals. For the steady state computed above, these two income distributions are given in Table 3.

Note that the difference in the mean incomes of next period's generation as produced by each community is relatively small despite the fact that per-capita education expenditures are very different across communities. This is due to the elasticity of future income with respect to education expenditures, which (despite the controversy) is a relatively small number.

B. Policy Experiment

In this section we determine the effects of switching to a public-education system in which there is no local financing. Rather, per-student expenditures on education are the same regardless of the community of residence

and the total level of expenditure is determined at the state (or national) level.²⁶ Of course, the manner by which revenue is raised is also important. We maintain the property tax as the tax instrument so as to keep the local-versus-state question starkly focused.

Formally, the stage game of Section II is modified so that in the second stage voting is over a single property tax rate with the understanding that the revenue proceeds will be spread so that expenditures per student are equal across communities.²⁷ It should be clear that in each period's equilibrium the price of housing must be equal across communities: since all individuals face the same tax rate and obtain the same quality of education independently of the community in which they live, no one would choose to reside in the community with the higher housing price.

We use the functional forms and parameter values from the calibration procedure described in the preceding section to determine the effects of the change in policy. It remains true that there is a unique equilibrium in each period, a unique stable steady state, and that the economy converges to the steady state. Table 4 displays some of the properties of the steady-state equilibrium.

²⁶ This is similar to the system of financing education of some European countries and some U.S. states.

²⁷ Note that no additional sources of education finance are allowed, i.e., no local supplements.

TABLE 4—STEADY STATE UNDER STATE FINANCING

$\lambda = (0.02, 0.06, 0.09, 0.21, 0.18, 0.13, 0.16, 0.10, 0.06)$
$t = 0.386, p = 1.327, q = 1.180$
$E/TC = 0.0560, H/TC = 0.1450$
mean income = 22.28, median income = 18.54

A comparison of the steady-state outcomes generated by the two systems yields several expected results. Since the economywide mean income and the income of the median voter lie between those of the individual communities, it is not surprising that, for example, spending per student in the steady state of the second system lies between the corresponding values for the two communities of the first system, as do the tax rate and the price of housing. Two results (not necessarily expected) are that both average income and education expenditures as a fraction of total consumption are greater under state financing. These will turn out to be central to our welfare results.

Our analysis also allows us to trace out the transition path between the two steady states. Properties of the transition will be important to the welfare calculation of the next subsection. For completeness, Table 5 shows the evolution of the tax rate, housing price, the quality of education, mean income, and E/TC . Period 0 is the steady state of the local finance equilibrium, and period 1 is the period in which state-level finance is introduced. In period 4 the economy with state finance effectively attains its steady state.

As can be noted from Table 5, the transition to the steady state is monotonic in all the relevant variables as of period 1. Compared to the local steady-state equilibrium (period 0), however, the transition entails a jump in E/TC as the median voter first chooses a high tax rate to increase spending on education by a large amount and then, as the income distribution shifts to the right and total income increases, the tax rate and consequently E/TC are chosen progressively smaller.²⁸

²⁸ The income level of the median voter remains unchanged throughout the transition. This need not always be the case and would generically not be so if a continuous income distribution were used.

TABLE 5—TRANSITION FROM PURE LOCAL-FINANCE SYSTEM TO STATE-FINANCE STEADY STATE

Period	t	p	q	μ	E/TC
1	0.4000	1.2840	1.1642	21.5632	0.0571
2	0.3874	1.3236	1.1799	22.2184	0.0561
3	0.3858	1.3273	1.1801	22.2760	0.0559
4	0.3858	1.3274	1.1801	22.2765	0.0559

Several researchers have noted that when California moved to a state-financing system following the *Serrano* decision and Proposition 13, the result was a substantial decrease in the share of income devoted to education (see Lawrence Picus [1991], for example). How can this be reconciled with the above prediction that a move from local to state financing implies an increase in the share of income devoted to education? As argued by Fernández and Rogerson (1995) in the context of a much simpler model, the key issue is to note that California did not switch from a pure local system to a state system, but rather from a foundation system to a state system. In a foundation system all districts receive a guaranteed amount from the state and are free to supplement this amount with local revenues. We show in that paper that a foundation system actually leads to greater spending than either extreme of a pure local or pure egalitarian state system.

We now turn to the question of the effect of this policy change on welfare.

C. Welfare Effects

It is clearly desirable to have some measure of the welfare effects associated with the change in the education-financing system. Since our economy consists of heterogeneous individuals and of different generations, this is not a straightforward exercise. In order to do so, we construct the following welfare measure. Under each financing system and for each period t , we compute the expected utility (EU) for a hypothetical individual whose income is a random draw from that period's income distribution. Thus, if λ_{it} is the fraction of the population with income y_i in period t , and

V_{it} is the utility of an individual with income y_i in that same period, then the expected utility in period t is given by:

$$(15) \quad EU_t = \sum_i \lambda_{it} V_{it}.$$

Henceforth we define EU_0 to be the value of EU in the steady state of the local-finance system.²⁹

We first examine the value of EU in the two steady states. Under the local system $EU_0 = -0.3197$; under the state system $EU = -0.3117$. While this indicates that under this welfare criterion the steady state of the state-finance system is preferred to the pure local system, in order to translate this difference in utility into a measure not affected by linear transformations of the utility function, we calculate the percent, Δ , by which the vector of income (y_1, y_2, \dots, y_9) would have to be reduced in the state-finance system to render the hypothetical individual indifferent between the two economies. This calculation holds prices, tax rates, and quality of education constant at their original equilibrium levels. More generally, to calculate the percentage income transfer that would be necessary to leave individuals from some generation t indifferent between the welfare level achieved in that time period and that obtained in the steady state of the local-finance system, we obtain Δ_t implicitly from:

$$(16) \quad \begin{aligned} EU_t(\Delta_t) - EU_0 \\ = \sum_i \lambda_{it} V(y_{it}(1 - \Delta_t), \pi_t, q_t) - EU_0 \\ = 0. \end{aligned}$$

Comparing the steady state under state financing with the steady state under local financing, the magnitude of the required decrease in income under state financing is 3.2 percent. That is, total income would have to be decreased by 3.2 percent *every* period in order for there to be indifference between the steady states of the two systems. This is a very large difference in welfare; these types of pol-

icy comparisons usually yield welfare costs of alternative policies of a fraction of 1 percent of total income.³⁰

Of course, a welfare comparison across steady states is not complete since the costs or benefits incurred along the transition path are ignored. We next examine, therefore, the welfare effects along the transition path to the new steady state. Table 6 shows the value of EU_t from period 0—the steady state under local financing—through period 4 (at which point the economy is at its steady-state equilibrium under the state-finance system). The third column gives, for each period t , the percentage Δ_t by which the vector of income would have to be changed in order to equate that period's EU_t to EU_0 (recall that prices, tax rate, and quality of education are kept at the equilibrium level attained in period t).³¹ Note that period 1 will have, *ceteris paribus*, a greater tendency to have a negative Δ associated with it since any change in income distribution will not be realized until the following period (i.e., the income distribution in period 1 is that from the steady state under local finance).³²

In order to obtain an overall welfare evaluation which includes the transition path, we need to assign a discount rate and the associated length of a period, τ . If each period is interpreted to be the productive life of an individual, 30 years seems a reasonable benchmark. The structure of our model, however, is such that an individual spends the same length of time going to school when young as being productive when old. Thus another reasonable alternative is a time period of 15 years. We explore the implications of both possibilities. Instead of arbitrarily assigning an annual discount factor, however, we ask at what rate must the future be discounted in

³⁰ Note that simply comparing steady-state income yields a very similar result: income is 3.3 percent greater under state financing. This is somewhat reassuring since concavity of the utility function alone implies that redistributive policies may increase aggregate welfare. The fact that steady-state income has increased by a comparable amount suggests that a large fraction of the gains are due to other sources.

³¹ Note that by our previous definition, $\Delta_0 = 0$.

³² See Benabou (1996) for a somewhat different tension that exists between long-run and short-run welfare.

²⁹ Note that this is simply a utilitarian social welfare function.

TABLE 6—WELFARE EFFECTS ALONG THE TRANSITION PATH

Period	EU_t	Δ_t
0	-0.3197	0
1	-0.3205	-0.3
2	-0.3124	2.9
3	-0.3117	3.2
4	-0.3117	3.2

order for both systems (including the transition to the state-financing system) to yield the same total utility. More formally, we find the β such that

$$(17) \quad \sum_{s=0}^{\infty} \beta^{\tau s} \Delta_{s+1} = 0.$$

For $\tau = 30$, this yields $\beta = 0.924$ or an implicit annual interest rate $r = 8.2$ percent. For any discount factor greater than 0.924, therefore, under the welfare criteria used it would be beneficial to switch to a state-finance system. Similarly, for $\tau = 15$ the implied annual interest rate required for indifference is $r = 17.2$ percent.

Note that the above welfare calculation did not take into account the welfare of the owners of housing (who receive the rental income). Total producer surplus from the housing market is easily computed in the two economies: Relative to its value in the steady state under local financing, it first decreases by 1.7 percent in period 1, and then increases by 2.89 percent in period 2, and thereafter by 3.33 percent in each subsequent period. The β required for indifference in this case is higher ($\beta = 0.930$ for $\tau = 30$). In any case producer surplus is a small fraction of total steady-state income (roughly 10 percent).

What is the source of the large overall welfare gain? By definition, any difference in welfare must be induced by changes in the V_i and/or changes in the λ_i . Thus, we must examine the change in utility derived from each income level's new c , h , and q bundle as well as the overall change in the income distribution.

Comparing across steady states for the two financing systems studied above, it turns out that each of the V_i is greater under local financing!³³ That is, for each given income level, utility under the steady state of the local-financing system is higher than that of the state-financing system. Since expected utility is higher under state financing, it must be the case, therefore, that favorable changes in the distribution of income (i.e., the λ_i 's) more than offset the decrease in the V_i 's. This points to a trade-off that is central to a comparison of a local- and a state-financing system. On the one hand, a local system has the ability to make individuals better off by allowing them greater scope to sort themselves into communities that more closely reflect their preferences *given* their income than does a state system that forces individuals to consume the same quantity of the publicly provided good. On the other hand, a state system may yield a better income distribution (in that higher output is generated) than a local system which generates greater heterogeneity in education expenditures. We now turn to a more detailed examination of these points.

Note first that the steady-state income distribution under state financing stochastically dominates that under local financing; in particular, λ_1 through λ_4 are greater under local financing whereas λ_5 through λ_9 are greater under state financing. The income distribution under state financing is characterized by a single parameter—the mean of the lognormal distribution (recall that the variance is constant). Thus, an explanation of the higher level of mean income should provide insight into the higher welfare achieved under the national financing system.

Although equation (13) allows the mean of log income to be either concave or convex in q , our finding of $\delta = -3.9$ implies substantial concavity. Because concavity of future income with respect to current educational spending turns out to be an important factor in our re-

³³ In fact, the V_i 's are also greater under the steady state of the local-financing system than in the first period after the reform. This implies that were this reform to be voted on, it would be rejected unanimously by all the old individuals.

sults, it is worthwhile to consider this issue in some detail. The relationship between next period's income y , and this period's spending on education q , for a given individual is given by:

$$(18) \quad \log(y) = m(q) + \varepsilon,$$

where $m(q)$ is given by equation (13) and ε is normal with mean 0, variance σ^2 , and is independently and identically distributed (i.i.d.) across individuals.³⁴ Although $\delta = -3.9$ implies that mean log income is concave in q , it does not necessarily follow that mean income is concave in q . Mean income is the expected value of the lognormal distribution and is given by $\exp(m(q) + \sigma^2/2)$. Hence, mean income is concave in q if and only if $m'(q) + m''(q) < 0$. This condition is satisfied if and only if $(1 + q)^\delta < (1 - \delta)/B$. One of the statistics we use in our calibration is the elasticity of future earnings with respect to spending. This can be computed analytically and is given by $Bq(1 + q)^{\delta-1}$. Matching a particular value for this elasticity does not impose any restriction on the concavity of next period's income with respect to this period's spending on education.³⁵ However, a second elasticity we match is the cross-community elasticity of spending with respect to community mean income. Subject to matching these two elasticities, our numerical work found no choices of B and δ for which mean income is not concave in spending.

What is the independent evidence on the concavity of this relationship? An alternative way to ask this question is what is the evidence on decreasing rates of return to spending on education. In a narrow sense the empirical literature is not very informative on this issue; given the controversy over whether resources matter for outcomes, there is no definitive

work on the extent to which this relationship is concave.³⁶

As just discussed, our calibration implies that mean income is concave in spending. It follows that holding total spending on education fixed, next period's mean income is greatest if these funds are divided equally across all students. Whereas equal division of funds is what occurs under state financing, under local financing students in C_2 receive roughly half the per-student expenditures as students in C_1 . To obtain an idea of how much this concavity matters, we calculate the income distribution that would result from distributing total steady-state expenditures on education in the local system equally across students. The mean of the resulting income distribution is 22.02, a gain of 2.1 percent over the mean of 21.56 that results from the pattern of educational expenditures found in the steady state under local financing and 63.9 percent of the total increase in mean income found in the steady state of the state system. Thus, there are large gains to be realized simply by spreading resources equally across all students. The remainder of the gain in mean income comes from the accumulated effects of this change in income and the increased education expenditures induced by a change in financing systems.

It may be thought that a substantial portion of the welfare increase is a consequence of concavity of preferences over q since our calibration implies that $v(q)$ is concave. Holding total spending on education constant, therefore, the average value of $v(q)$ is maximized by a constant q across communities. A simple calculation, however, indicates that the quantitative magnitude of this effect is small. In particular, using the steady-state equilibrium values under local financing yields

³⁴ Recall that we actually use a discrete approximation to this relation.

³⁵ This should be contrasted with a simpler specification that is commonly employed, namely, $y = Aq^\theta \varepsilon$, where ε is an i.i.d. income shock. With this specification, the parameter θ is the elasticity of future earnings with regard to educational expenditures, and the value of this elasticity completely determines the concavity of income with respect to education spending. See Fernández and Rogerson (1997b) for more details.

³⁶ At a broader level, there is some evidence to suggest that the rate of return to schooling decreases as schooling increases. Willis (1986) reviews much of the relevant literature, and finds evidence to support this. Specifically, he finds rates of return equal to 17–22 percent for lower education, 15–16 percent for high school, 12–13 percent for college, and 7 percent for graduate school. While these numbers are not directly relevant for the relationship discussed above, they do support the notion that as educational resources are increased, the rate of return decreases.

0.0008 as the amount by which $v(\rho^*q_1^* + (1 - \rho^*)q_2^*)$ exceeds $\rho^*v(q_1^*) + (1 - \rho^*)v(q_2^*)$, which is only about 10 percent of the difference in steady-state expected utilities for the two financing systems.

We now turn to a closer examination of the trade-off between local- and state-financing systems via the use of two illustrative examples.

D. Two Examples

The previous discussion of welfare effects highlighted two opposing forces central to a comparison of local- and state-financing systems. On the one hand, local finance permits heterogeneous agents to obtain bundles closer to their preferred ones. On the other hand, the equalization of expenditures across students that occurs in a state system results in greater mean income. In our benchmark model the second effect is dominant. Here we present two examples to show that this outcome is a result of the particular parameter values generated by our calibration procedure and is not inherent to the structure of the model. These examples may also help to illustrate the nature of the trade-off described above.

Table 7 displays parameter values (where different from the benchmark model) and some selected statistics for the steady-state allocations under local financing for the two examples and for the benchmark. As the table indicates, both examples are not acceptable from the perspective of our calibration procedure. Most importantly, in example 1, $\varepsilon_{q,\mu}$ is too high and in example 2, $\varepsilon_{m,q}$ is too low. Our focus is on the predictions of these two examples for steady-state welfare gains associated with a change from local finance to state finance. These are reported on the last row of the table. Expressed as before in terms of output, Δ , the gains are -6.2 and $+0.27$ percent for examples 1 and 2 respectively.

Table 8 presents two additional pieces of information useful for interpreting the above differences in welfare predictions. First it lists preferred tax rates in the steady state under state financing by income level for the benchmark

TABLE 7—SELECTED FEATURES
UNDER LOCAL FINANCING

	Benchmark model	Example 1	Example 2
δ	-3.9	0.5	-8
B	8	0.23	35
y_0	3.0	2.0	2.9
α_q	0.03	0.05	0.20
μ	21.56	18.16	21.79
$EITC$	0.0545	0.0852	0.0556
$\varepsilon_{q,\mu}$	0.616	3.12	0.35
$\varepsilon_{m,q}$	0.19	0.15	0.03
Δ	3.2	-6.2	0.27

model and the two examples. This provides some indication of the extent to which individuals desire different bundles of goods.³⁷ Preferred tax rates exhibit the smallest range in example 2 and the greatest range in example 1. The second piece of information provided is the percent change in mean income (percent μ) that would result if the resources devoted to education in the local-financing steady state were spread equally across all students (as in the earlier discussion). This figure provides some indication of the potential gains from equalizing expenditures. Note that this number is largest in the benchmark model and smallest in example 2.

An explanation of the contrasting results for welfare gains in the three cases is as follows. Example 2 is a case where spending on education is not very important (as evidenced by the small value of $\varepsilon_{m,q}$). Consequently, neither of the two factors mentioned above is particularly significant and the overall welfare gain is also small. Example 1 is a case where heterogeneity is quite important. Thus, although there are sizable gains to be had simply by smoothing expenditures across students, these are outweighed by the gains associated with allowing individuals to sort themselves into different communities. Relative to example 1, the benchmark model reverses the relative

³⁷ Note that this is a rough indication since both preferences and technology differ in the three cases.

TABLE 8—PREFERRED TAX RATES UNDER STATE FINANCING AND PERCENT μ

	Benchmark model	Example 1	Example 2
y_1	0.22	0.00	0.30
y_2	0.30	0.08	0.35
y_3	0.32	0.16	0.36
y_4	0.35	0.32	0.38
y_5	0.39	0.51	0.40
y_6	0.41	0.68	0.42
y_7	0.44	0.93	0.44
y_8	0.49	1.31	0.46
y_9	0.53	1.81	0.48
Percent μ	2.1	1.0	0.2

magnitudes of the two effects yielding a large overall welfare improvement.

IV. Sensitivity Analysis and Alternative Comparisons

Our calibration exercise relies on several estimates obtained from empirical work. Because the empirical studies often suggest a range of estimates rather than a single value, it is of interest to check the sensitivity of our policy analysis to the use of alternative values in the calibration exercise. Furthermore, it is also of interest to see what our model implies for some statistics not used in our calibration exercise.

A. Sensitivity to Alternative Parameter Values

In all the exercises that follow, the model's parameters are recalibrated, i.e., parameters are chosen so that the model's steady state matches the appropriate set of statistics. To focus the discussion, we consider only the effect of these alternatives on the steady-state welfare calculation carried out in Section III, subsection C.

We begin with a brief discussion of three variations that we found to have virtually no effect on the magnitude of the steady-state welfare gain: changes in the preference parameter γ and in the two housing price elasticities.

Recall that γ was set to 0 in the benchmark specification. The results are very similar when γ is set to values of 0.5, 0.25, -1, and -5. Similarly, we found no significant effect of changing the values of the price elasticities of housing used in the calibration. For the demand elasticity we used a value of -0.93 (which corresponds to the upper range of estimates), and for the supply elasticity we used values ranging from $1/3$ to 3.

In contrast to the above, we found that variations in $\varepsilon_{m,q}$ and $\varepsilon_{q,\mu}$ have a significant impact on the magnitude of the steady-state welfare gains predicted by our model. Table 9 presents the results for several alternative values of these two elasticities. (In each case the table shows the parameter values that differ from those of the benchmark specification.) In the interest of space we do not include any of the other summary statistics for the steady-state equilibria, but note that in all cases these values are similar to those for the benchmark model.

From the table we note that although changes in $\varepsilon_{q,\mu}$ result in a sizable range of associated welfare gains, in all cases the welfare gain is substantial. Furthermore, welfare gains significantly larger than those reported in our welfare section, are apparently plausible. As for the alternative values for $\varepsilon_{m,q}$, these are not chosen in accord with those suggested by any specific empirical work. Nonetheless, given the controversy in this area, we think it informative to indicate the sensitivity of our results to this value. For the range of values displayed in the table, the steady-state welfare gain appears to be roughly linear in this elasticity. Note that even using a value almost 50 percent lower than our chosen estimate, the welfare gains are still sizable.

B. Some Additional Comparisons

In addition to the sensitivity analysis performed above, an additional check on the reasonableness of our calibrated model can be obtained by contrasting the income distribution and the intergenerational mobility generated by our model with that observed in reality as well as with some alternative measures of the rate of return to education. These are all

TABLE 9—SENSITIVITY ANALYSIS

$\varepsilon_{q,p}$	δ	B	y_0	a_q	Δ	
0.62	-3.9	8.00	3.01	0.034	3.20	
1.28	-1	0.95	3.37	0.030	5.67	
0.92	-2	2.00	3.135	0.032	4.48	
0.74	-3	4.30	3.052	0.030	4.10	
0.51	-5	18.5	2.985	0.032	2.90	
0.39	-7	90.0	2.950	0.035	2.23	
$\varepsilon_{m,q}$	δ	B	y_0	a_q	Δ	r
0.62	-3.9	8.00	3.01	0.034	3.20	8.6
0.12	-4.5	7.70	2.95	0.053	1.60	5.4
0.31	-3.8	12.25	3.105	0.021	5.69	12.2

measures that our model did not calibrate to directly.

The calibration ensures that the mean and median income in the model's steady state are approximately equal to their counterparts in the U.S. data. The distribution of income from the 1980 U.S. Census is given by (0.07, 0.06, 0.07, 0.15, 0.15, 0.14, 0.19, 0.11, 0.06). As is well known, the lognormal distribution does a good job of accounting for the observed income distribution except that it does not have enough mass in the tails. Not surprisingly, therefore, comparing with the λ in Table 1, the same is true of the model's steady-state income distribution.

The intergenerational mobility implied by the steady-state equilibrium of the model are summarized by the numbers in Table 10 and contrasted with averages obtained for the United States by David J. Zimmerman (1992) which are presented in parentheses. Note that while our model does fairly well for the mobility numbers in the second and third quartile, it produces a smaller probability of a child ending up in the top quarter given that the parent is in that quarter and likewise a smaller probability of remaining in the bottom quarter given that the parent is in that quarter. This is probably in large part due to the fact that we only have two communities and use a lognormal distribution to approximate the income distribution generated by the quality of education in each community. A larger number of

TABLE 10—INTERGENERATIONAL INCOME MOBILITY
(NUMBERS IN PARENTHESES ARE FROM ZIMMERMAN, 1992)
PARENT'S INCOME QUARTILE

		Top	Second	Third	Bottom
Child's Income Quartile	Top	0.31 (0.42)	0.25 (0.26)	0.22 (0.20)	0.22 (0.11)
	Second	0.23 (0.34)	0.24 (0.24)	0.25 (0.23)	0.25 (0.21)
	Third	0.24 (0.16)	0.25 (0.27)	0.26 (0.31)	0.26 (0.33)
	Bottom	0.22 (0.09)	0.25 (0.24)	0.27 (0.26)	0.27 (0.35)

communities would give wealthier parents access to a higher q (and thus their children a greater probability of being likewise wealthy) and the opposite would hold for poorer parents.^{38,39}

An additional piece of information that can be computed using the steady-state allocations is the implicit rate of return to expenditures on education. In the steady state, C_1 spends an additional $q_1 - q_2$ per student on education. This leads to a gain in mean income of $\mu_{c1} - \mu_{c2}$, where $\mu_{c,j}$ is the mean future income for children who go to school in community j . Assuming that a period lasts τ years, the implied annual rate of return r satisfies:

$$(19) \quad (1 + r)^\tau = \frac{\mu_{c1} - \mu_{c2}}{q_1 - q_2}.$$

For a time period of 30 years, using the appropriate steady-state values yields $r = 0.0422$ whereas for $\tau = 15$, $r = 0.086$.

There is a fairly large literature that attempts to determine the rate of return to investment in human capital, in particular, the return to an additional year of schooling (see, for example,

³⁸ Note that under state financing, Table 10 would have 0.25 for all its entries since parental income does not affect the child's income under a state system. Hence there is greater intergenerational mobility between the bottom and the top quartiles under a state-financing system.

³⁹ Higher values of either $\varepsilon_{m,q}$ or $\varepsilon_{q,p}$ in the calibration also make the model's predictions closer to Zimmerman's numbers.

Becker, 1975). Annual returns of between 4 and 9 percent are in the lower half of the range found in this literature, where the typical range is 5–15 percent. Moreover, the estimates of Willis (1986) for primary and secondary education suggest that the upper part of this range is more appropriate. Although our calibration procedure does not attempt to match this rate of return, it is obviously closely related to $\varepsilon_{m,q}$, which is defined as $[\log(\mu_{c1}) - \log(\mu_{c2})] / [\log(q_1) - \log(q_2)]$. Thus one possibility is to calibrate to a larger $\varepsilon_{m,q}$ yielding higher implied annual rates of return. The final column in Table 9 shows the annual rates of return based on $\tau = 15$. If $\tau = 30$ is used, the rates of return are roughly half as large. Based on this evidence, the value of $\varepsilon_{m,q}$ used in the calibration does not seem extreme.

V. Concluding Remarks

This paper argues that dynamic considerations are an important element in evaluating alternative school-finance systems. To make this argument, we developed a tractable dynamic multicommodity model and calibrated it to U.S. data. We used the calibrated model to evaluate the consequences of reforming the public-education finance system from one of pure local finance to a system in which education is financed at the state level and expenditures per student are equal across communities.

We analyzed the effects of such a reform on allocations and welfare, both across steady states and along the transition path. Our findings indicate a substantial welfare gain associated with this change in policy. In our benchmark model the steady-state welfare gain associated with the state-finance system is over 3 percent of total income.

Some simplifying features of the model should be kept in mind when interpreting the above welfare gain.⁴⁰ First, we assumed that there are only two communities. In a simpler setting, however, Fernández and Rogerson (1997b) assumed perfect separation of individuals and found similar welfare gains. Second, our analysis assumes that all parents send

their children to public schools. While currently less than 10 percent of children attend private schools in the United States, it is possible that a move to a state-finance system would increase this proportion and thereby diminish public support for public expenditure on education.⁴¹ Third, we assume that the quality of education is only affected by spending per student; in particular, we abstract from any peer effects and assume that parental characteristics do not influence educational outcomes other than through spending on education. Fourth, this welfare gain presumably overstates the potential gains from reform facing a state whose educational-finance system is somewhere between the extremes of local and national financing.⁴² Future work should focus on evaluating how incorporating these factors influences the evaluation of public-education finance systems and extending this work to mixed-finance systems.

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⁴¹ Recent evidence from California provides some indication of a reasonable magnitude. Following its reform, the share of students in private school in California rose from 8 percent to 11 percent.

⁴² In the United States, local spending accounts for roughly 45 percent of all spending on public education. Potential benefits from reforms depend on both the fraction of total expenditures accounted for by state aid and on the rules which govern its allocation. A system whereby state aid simply matches local spending dollar for dollar is obviously quite different from one in which aid is primarily targeted to lower-income communities. The framework developed here can also be used to analyze systems which involve a mix of local and state financing.

⁴⁰ It should be noted that our analysis does not recommend the centralized provision of education but rather the centralization of its financing.

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