1 Model Tricks in HSV

Goal: Build a model

- in which people make labor supply and skill investment choices
- which generates realistic heterogeneity in hours, earnings and consumption
- with a reasonable approximation to the US tax and transfer system
- which is entirely tractable

Use the model to assess

- Whether the current system is optimally progressive
- How choices for redistribution and government spending interact
- How should government respond to rising inequality?

Tricks to preserving tractability

- Use special preferences: quadratic, or CARA?
- Abstract from idiosyncratic risk?
- Our approach: standard preferences but a carefully chosen idiosyncratic risk process and asset market structure

Start with an endowment economy with wealth in zero net supply
Agents trade a risk-free bond and can borrow and lend
Transitory shocks: those with a good shock save, those with a bad shock borrow
Permanent shocks: not so obvious (recall the Barro tax smoothing example)

2 Constantinides and Duffie 1994 No Bond Trade Result

Suppose agents solve

$$\max \sum_{t=0}^{\infty} \beta^t E\left[\frac{c_t^{1-\gamma}}{1-\gamma}\right]$$

s.t.

$$\begin{aligned} c_t + a_{t+1} &= y_t + (1+r)a_t \\ a_{t+1} &\geq \underline{a} \\ a_0 &= 0 \end{aligned}$$

where

$$\begin{aligned}\log y_{t+1} &= \log y_t + \omega_{t+1} \\ y_{t+1} &= y_t \exp(\omega_{t+1})\end{aligned}$$

$$\omega_{t+1}\,\tilde{}\,N\left(\mu,\sigma^2\right)$$

Market clearing for this economy is

$$\begin{aligned}\int c_{it}di &= \int y_{it}di \\ \int a_{i,t+1}di &= 0 \text{ for all } t\end{aligned}$$

The FOC for the household is

$$c_t^{-\gamma} = \beta(1+r)E_t\left[c_{t+1}^{-\gamma}\right]$$

Let us guess and verify that there is an equilibrium in which $c_{it} = y_{it}$ for all i and t.

Can households possibly be happy with that choice?

Plug conjectured allocation into their FOC

$$\begin{aligned}y_t^{-\gamma} &= \beta(1+r)E_t\left[y_{t+1}^{-\gamma}\right] \\ y_t^{-\gamma} &= \beta(1+r)E_t\left[y_t^{-\gamma}\exp(\omega_{t+1})^{-\gamma}\right] \\ 1 &= \beta(1+r)E_t\left[\exp(\omega_{t+1})^{-\gamma}\right]\end{aligned}$$

Note that $E_t\left[\exp(\varepsilon_{t+1})^{-\gamma}\right]$ is independent of y_t. Thus, as long as

$$1+r = \frac{1}{\beta E_t\left[\exp(\omega_{t+1})^{-\gamma}\right]}$$

all agents will be saving / borrowing optimally when $c_{it} = y_{it}$.

This is a trick we borrowed from Constantinides and Duffie (1994). We will show that the trick still works with labor supply + progresive taxes etc.

What is $E_t\left[\exp(\omega_{t+1})^{-\gamma}\right]$? We know that

$$\log(\exp(-\gamma\omega_{t+1}))\,\tilde{}\,N\left(-\gamma\mu,\gamma^2\sigma^2\right)$$

So $\exp(-\gamma\varepsilon_{t+1})$ is log-normally distributed. So we can use the formula for the expectation of a log-normal variable which is

$$E\left[\exp(-\gamma\omega_{t+1})\right] = \exp\left(-\gamma\mu + \frac{\gamma^2\sigma^2}{2}\right)$$

So we have

$$\beta\exp\left(-\gamma\mu + \frac{\gamma^2\sigma^2}{2}\right) = \frac{1}{1+r}$$

Define $\rho \equiv \frac{1-\beta}{\beta}$, so $\beta \equiv \frac{1}{1+\rho}$

$$\frac{1}{1+\rho} \exp\left(-\gamma\mu + \frac{\gamma^2\sigma^2}{2}\right) = \frac{1}{1+r}$$

Take logs of both sides, using the approximation

$$\log \frac{1}{1+\rho} = \log 1 - \log(1+\rho) \approx -\rho$$

$$r = \rho + \gamma\mu - \frac{\gamma^2\sigma^2}{2}$$

Note that expected income growth is

$$E\left[\frac{y_{t+1}}{y_t}\right] = \exp\left(\mu + \frac{\sigma^2}{2}\right) \approx \mu + \frac{\sigma^2}{2} \equiv g$$

So we have

$$\begin{aligned} r &= \rho + \gamma g - \gamma(1+\gamma)\frac{\sigma^2}{2} \\ \underbrace{(1+\gamma)\frac{\sigma^2}{2}}_{\text{precautionary saving}} + \underbrace{\frac{r-\rho}{\gamma}}_{\text{intertemporal saving}} &= \underbrace{g}_{\text{growth borrowing}} \end{aligned}$$

3 Perfect Insurance Against Transitory Shocks

This is fine, but this is a unit root process for idiosyncratic productivity risk

We know there are also more transitory shocks

Won't introducing those break the no-trade result?

Yes in a Huggett economy

No if we introduce explicit insurance against transitory shocks

Our approach

$$\begin{aligned} \log y &= \alpha + \varepsilon \\ \alpha_{t+1} &= \alpha_t + \omega_{t+1} \\ & \omega_{t+1} \tilde{} N(-\frac{v_\omega}{2}, v_\omega) \\ & \varepsilon_{t+1} \tilde{} N(-\frac{v_\varepsilon}{2}, v_\varepsilon) \end{aligned}$$

Complete set of state-contingent claims indexed to ε_{t+1}

Timing is: (1) ω_t realized, (2) buy insurance against ε_t, (3) ε_t realized, claims pay off, make labor supply choice and buy bonds a_{t+1} to carry into next period.

No bond trade result survives

4 Tax and Transfer Function and Labor Supply

Let's now look at the labor supply and tax specification tricks

Consider a static model

Suppose we have the following HH problem

$$\max\left\{\frac{c^{1-\gamma}}{1-\gamma}-\frac{n^{1+\sigma}}{1+\sigma}\right\}$$

subject to

$$\begin{aligned} c &= wn - T(wn) \\ T(wn) &= wn - \lambda(wn)^{1-\tau} \end{aligned}$$

Substituting constraints into the objective

$$\max_n\left\{\frac{\left(\lambda(wn)^{1-\tau}\right)^{1-\gamma}}{1-\gamma}-\frac{n^{1+\sigma}}{1+\sigma}\right\}$$

The FOC is

$$\left(\lambda(wn)^{1-\tau}\right)^{-\gamma}\lambda(1-\tau)(wn)^{-\tau}w = n^{\sigma}$$

$$n = \left[\lambda^{1-\gamma}w^{1-\gamma(1-\tau)-\tau}(1-\tau)\right]^{\frac{1}{\sigma+\tau+\gamma(1-\tau)}}$$

Suppose we set $\gamma = 1$

Then

$$n = (1-\tau)^{\frac{1}{1+\sigma}}$$

Let's explore the tax function a bit:

$$\begin{aligned} T(y) &= y - \lambda y^{1-\tau} \\ T'(y) &= 1 - \lambda(1-\tau)y^{-\tau} \\ \frac{T(y)}{y} &= 1 - \lambda y^{-\tau} \end{aligned}$$

$\tau = 0 =>$ proportional tax

$$\begin{aligned} T(y) &= (1-\lambda)y \\ T'(y) &= 1-\lambda \\ \frac{T(y)}{y} &= 1-\lambda \end{aligned}$$

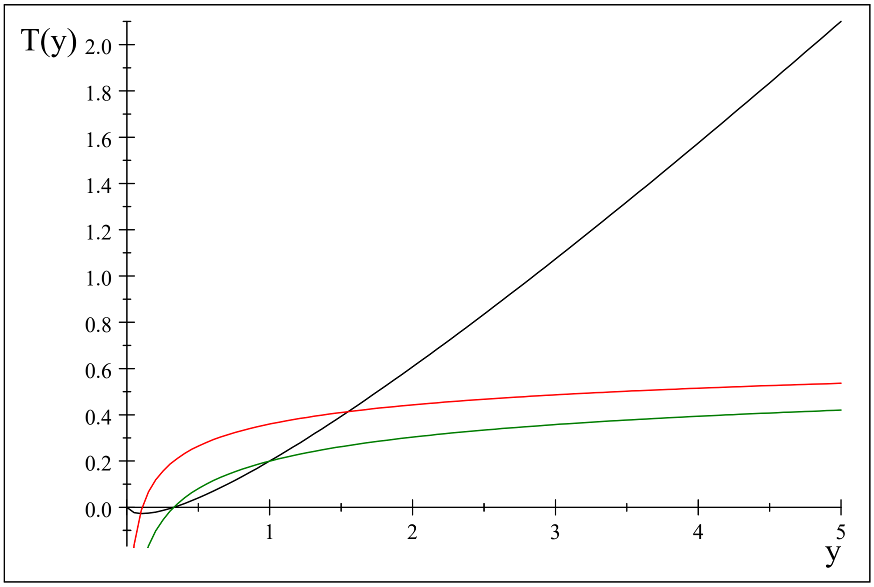
$\tau > 0 =>$ marginal tax rates increase with income $=>$ tax system is progressive

$\tau < 0 =>$ marginal tax rates decline with income $=>$ tax system is regressive

Disposable income $\tilde{y}$ has a log-linear relation to pre-tax income y

$$\begin{aligned} \tilde{y} &= y - T(y) = \lambda y^{1-\tau} \\ \log(\tilde{y}) &= \log\lambda + (1-\tau)\log y \end{aligned}$$

Here we set $\lambda = 0.8$ $\tau = 0.2$ and plot $T(y)$ (in black), $T'(y)$ (in red) $\frac{T(y)}{y}$ (in green)



Note that

$$\begin{aligned} c &= \lambda (wn)^{1-\tau} \\ &= \lambda w^{1-\tau} (1-\tau)^{\frac{1-\tau}{1+\sigma}} \end{aligned}$$

so

$$\frac{d \log c}{d \log w} = (1-\tau)$$

i.e., tax progressivity reduces pass through from wages to consumption

5 Optimal Tax Progressivity in a Rep Agent Setting

Warm up to optimal tax problem

Consider a representative agent version of the model

Rep agent values C, N and G according to

$$u(C, N, G) = \log C - \frac{N^{1+\sigma}}{1+\sigma} + \chi \log G$$

Planner restricted to HSV tax and transfer class

So rep agent budget constraint is

$$C = \lambda (WH)^{1-\tau}$$

We know the solution

$$N(\tau) = (1-\tau)^{\frac{1}{1+\sigma}}$$

$$C = \lambda W^{1-\tau}(1-\tau)^{\frac{1-\tau}{1+\sigma}}$$

The govt. budget constraint is

$$G = WN - \lambda (WN)^{1-\tau}$$

Define

$$g = \frac{G}{WN}$$

We have

$$\begin{aligned} (1-g)WN &= \lambda (WN)^{1-\tau} \\ \lambda &= (1-g)(WN)^{\tau} \end{aligned}$$

So we can write utility, as a function of the policy parameters τ and g, as

$$\begin{aligned} u(C,N,G) &= \log\left((1-g)(WN(\tau))^{\tau} W^{1-\tau} N(\tau)^{1-\tau}\right) - \frac{1-\tau}{1+\sigma} + \chi \log(gWN(\tau)) \\ &= \log\left((1-g)WN(\tau)\right) - \frac{1-\tau}{1+\sigma} + \chi \log(gWN(\tau)) \end{aligned}$$

Take FOCs wrt g and τ

$$\frac{-1}{1-g} + \frac{\chi}{g} = 0 => g = \frac{\chi}{1+\chi}$$

$$\begin{aligned} (1+\chi)\frac{\frac{\partial N(\tau)}{\partial \tau}}{N(\tau)} + \frac{\tau}{1+\sigma} &= 0 \\ -(1+\chi)\frac{\frac{1}{1+\sigma}(1-\tau)^{\frac{1}{1+\sigma}-1}}{(1-\tau)^{\frac{1}{1+\sigma}}} + \frac{1}{1+\sigma} &= 0 \\ -(1+\chi)(1-\tau)^{-1} + 1 &= 0 => \tau = -\chi \end{aligned}$$

6 Adding Skill Investment

Suppose people can invest in skills: costly to build skills, but payoff in terms of higher future wages

How big that payoff is will depend on progressivity of tax system

Suppose utility cost of skill investment is

$$-\frac{1}{(\kappa_i)^{\frac{1}{\psi}}}\frac{s^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}}$$

where s is a continuous measure of skills (e.g., years of education) and κ_i is an idiosyncratic measure of ability (more able individuals find it cheaper to build skills)

Assume κ_i is exponentially distributed $\kappa_i \tilde{} Exp(1)$

Let $m(s)$ denote the equilibrium density for skills

Assume output is CES aggregator of different skill types

$$Y = \left[\int_0^\infty m(s)^{\frac{\theta-1}{\theta}} ds\right]^{\frac{\theta}{\theta-1}}$$

where $\theta \in (1, \infty)$ is the elasticity of substitution between skill types

Assume competitive labor markets: wages are marginal products

$$p(s) = \frac{\partial Y}{\partial m(s)} = Y^{\frac{1}{\theta}} m(s)^{\frac{-1}{\theta}}$$

where $p(s)$ is the price (wage) for a unit of labor of skill type s

Suppose (abstracting from labor supply)

$$c(s) = \lambda p(s)^{1-\tau}$$

Optimal skill investment: equate marginal cost and marginal gain

$$\begin{aligned}
\frac{s^{\frac{1}{\psi}}}{\kappa^{\frac{1}{\psi}}} &= \frac{\partial u}{\partial c}\frac{\partial c}{\partial p(s)}\frac{\partial p(s)}{\partial s} \\
\frac{s^{\frac{1}{\psi}}}{\kappa^{\frac{1}{\psi}}} &= \frac{1}{\lambda p(s)^{1-\tau}}\lambda(1-\tau)p(s)^{-\tau}\frac{\partial p(s)}{\partial s} \\
\frac{s^{\frac{1}{\psi}}}{\kappa^{\frac{1}{\psi}}} &= \frac{(1-\tau)}{p(s)}\frac{\partial p(s)}{\partial s} \\
&= (1-\tau)\frac{\partial \log p(s)}{\partial s}
\end{aligned}$$

$$s = \left[(1-\tau)\frac{\partial \log p(s)}{\partial s}\right]^{\psi} \kappa \tag{1}$$

We also know that

$$\log p(s) = \frac{1}{\theta}\log Y - \frac{1}{\theta}\log m(s) \tag{2}$$

So we have two sets of equations involving $p(s)$, both of which must be satisfied

In general solving for the $p(s)$ function is a difficult fixed point problem:

need to know $p(s)$ to figure out optimal skill investment from eq. 1, which determines $m(s)$

But need to know $m(s)$ to figure out skill prices from eq. 2

And we are solving for a function, not a single price

Trick: guess and verify log skill prices are linear in skills

$$\log p(s) = \pi_0 + \pi_1 s$$

Then skill investment problem gives

$$s = (1-\tau)^{\psi} \pi_1^{\psi} \kappa$$

Now if skills are proportional to κ, the skill distribution will be exponential, like the κ distribution

κ is exponential with parameter η, so s is exponential with parameter $\zeta = \eta\pi_1^{-\psi}(1-\tau)^{-\psi}$

i.e.

$$m(s) = \zeta \exp(-\zeta s)$$

So

$$\begin{aligned} \log p(s) &= \frac{1}{\theta}\log Y - \frac{1}{\theta}\log m(s) \\ &= \frac{1}{\theta}\log Y - \frac{1}{\theta}(\log\zeta - \zeta s) \end{aligned}$$

So we have

$$\begin{aligned} \pi_1 &= \frac{\zeta}{\theta} \\ &= \frac{\eta\pi_1^{-\psi}(1-\tau)^{-\psi}}{\theta} \end{aligned}$$

i.e.,

$$\pi_1^{1+\psi} = \left(\frac{\eta}{\theta}\right)(1-\tau)^{-\psi}$$

So increasing τ increases the pre-tax skill premium

So skill investment rule is

$$\begin{aligned} s &= \pi_1^{\psi}(1-\tau)^{\psi}\kappa \\ s &= \left(\left(\frac{\eta}{\theta}\right)(1-\tau)^{-\psi}\right)^{\frac{\psi}{1+\psi}}(1-\tau)^{\psi}\kappa \\ &= \left(\frac{\eta}{\theta}\right)^{\frac{\psi}{1+\psi}}(1-\tau)^{\frac{\psi}{1+\psi}}\kappa \end{aligned}$$

so higher tax progressivity reduces skill investment, and ψ controls the elasticity. Note that this is why the skill premium π_1 is increasing in τ

Note that

$$\begin{aligned}
\log p(s) &= \pi_0 + \pi_1 s \\
&= \pi_0 + \pi_1^{1+\psi}(1-\tau)^{\psi}\kappa \\
&= \pi_0 + \left(\frac{\eta}{\theta}\right)(1-\tau)^{-\psi}(1-\tau)^{\psi}\kappa \\
&= \pi_0 + \left(\frac{\eta}{\theta}\right)\kappa
\end{aligned}$$

So the variance of log skill prices is independent of tax progressivity

Why is that?

A higher value for τ reduces skill investment => less inequality in s, which is a force for less inequality

But a higher value for τ increases π_1 => more inequality in wages for a given distribution of skills, a force for more inequality

These two forces exactly offset