Homework 4, Econ 606

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Due in class, Tuesday March 28th

Consider the following economy

Each period a continuum of mass $(1-\delta)$ agents are born at age 0

Agents survive from age a to $a+1$ with constant probability δ.

The total population is

$$(1-\delta)(1+\delta+\delta^2+...)=1$$

Assume the environment is stationary, so we need not worry about time subscripts

Agents maximize expected utility, which is given by

$$E\sum_{a=0}^{\infty}(\beta\delta)^a u(c_a)$$

where

$$u(c)=\frac{c^{1-\gamma}}{1-\gamma}$$

Agents are subject to idiosyncratic wage shocks that are *iid* across agents. Agents supply one unit of time per period. The initial wage at age zero is lognormally distributed:

$$\ln(w_0)=\alpha_0 \sim N\left(-\frac{v_0}{2},v_0\right)$$

Wages subsequently evolve according to

$$\begin{aligned}\alpha_{a+1} &= \alpha_a+\omega_{a+1} \qquad a\geq 0\\ &\omega_{a+1} \sim N\left(\mu-\frac{v_\omega}{2},v_\omega\right)\end{aligned}$$

where the actual (level) wage is $w_a=\exp(\alpha_a)$.

The market structure is as follows. Agents are endowed with zero wealth at birth. Then they can trade bonds at a constant price q. They cannot trade prior to drawing α_0. The generic budget constraint is

$$\begin{aligned}c_a+qb_{a+1} &= b_a+w_a \qquad a\geq 0\\ b_0 &= 0\end{aligned}$$

There is a borrowing constraint,

$$b_{a+1} \geq \phi$$

Bonds are in zero net supply.

1. Suppose, to start that $\phi = 0$. Argue that in equilibrium it must be the case that

$$\begin{aligned} b_a &= 0 \qquad a \geq 0 \\ c_a &= w_a \qquad a \geq 0 \end{aligned}$$

   Argue (intuitively) that any value for q above a certain threshold is an equilibrium.

2. Suppose now that $\phi < 0$. Show that there is an equlibrium with allocations as in part (1) and derive an expression for the (now unique) equilibrium value for q.[1]

3. We could imagine cross-sectional consumption inequality in this economy increasing because of (i) an increase in μ (which would increase inequality across age groups), (ii) because of an increase in v_0, or (iii) because of an increase in v_ω. Assuming $\gamma > 1$, characterize the effects of these three different changes on the equilibrium interest rate $r = \frac{1-q}{q}$ and give some intuition.

4. Derive an expression for expected lifetime utility for a newborn agent who has yet to draw any shocks.

5. Define the (steady state) welfare effect of a change in the wage process from (μ, v_0, v_ω) to $(\widehat{\mu}, \widehat{v}_0, \widehat{v}_\omega)$ as the value for λ that satisfies

$$E_{|(\mu,v_0,v_\omega)} \sum_{a=0}^{\infty} (\beta\delta)^a u(c_a(1+\lambda)) = E_{|(\widehat{\mu},\widehat{v}_0,\widehat{v}_\omega)} \sum_{a=0}^{\infty} (\beta\delta)^a u(c_a)$$

   This welfare measure basically asks "by what percentage amount would one have to increase consumption in every date and state in an economy with (μ, v_0, v_ω) to leave an agent indifferent between being born into the (μ, v_0, v_ω) economy versus being born into the $(\widehat{\mu}, \widehat{v}_0, \widehat{v}_\omega)$ economy." Assume $\beta = 0.96$, $\delta = 0.98$, $\gamma = 2$, $\mu = 0.02$, $v_0 = 0.2$, $v_\omega = 0.02$. Using your answer to (4) compute values for λ corresponding to

   (a) $\mu \to \widehat{\mu} = 0.03$
   (b) $v_0 \to \widehat{v}_0 = 0.3$
   (c) $v_\omega \to \widehat{v}_\omega = 0.03$

[1] Here is a useful trick which I tried to explain in class that will be useful for this problem. Suppose $\omega \tilde{} N(\mu, \sigma^2)$. Then $E[\exp(\omega)] = \exp(\mu + \frac{\sigma^2}{2})$. It follows directly that for some constant κ, $\kappa\omega \tilde{} N(\kappa\mu, \kappa^2\sigma^2)$, so $E[\exp(\kappa\omega)] = E[\exp(\omega)^\kappa] = \exp(\kappa\mu + \frac{\kappa^2\sigma^2}{2})$.