Homework 5, Econ 606

Jonathan Heathcote

Due in class, Tuesday April 4th

Consider the following consumption-savings problem:

An individual faces two possible realizations for their wage in each period, $w \in \{w_l, w_h\}$ where $0 < w_l < w_h$.

At time zero, w_0 may be high or low with equal probability.

In subsequent periods, wages evolve stochastically according to a Markov process defined by the transition probability matrix π.

The individual must choose consumption c_t, savings in a non-contingent bond a_{t+1} and hours worked, n_t at each date t to maximize expected lifetime utility, where utility associated with an allocation $\{c_t, n_t\}_{t=0}^{\infty}$ is given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$$

$$u(c_t, n_t) = \frac{1}{1-\gamma}\left[c_t - \frac{\psi n_t^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}}\right]^{1-\gamma}$$

where $\psi, \varepsilon, \gamma > 0$ (these are known as Greenwood, Hercowitz and Huffmann preferences)

Suppose that initial wealth at time zero is given by a_0 and that borrowing is not permitted: $\phi = 0$. Hours and consumption must be non-negative (there is no upper bound on hours). Suppose that the net interest rate on bonds is r. Suppose that $\beta(1+r) < 1$

1. Formulate the individual's maximization problem recursively
2. Write down the first order conditions
3. What is the elasticity of hours worked with respect to the wage?

   Consider a version of this model with a very large number (continuum) of agents, where each agent draws wage shocks independently. Imagine that a planner chooses allocations. The planner cannot transfer resources across time.

4. Define the planner's problem
5. Characterize the allocations that solve the planner's problem.

(a) Does the planner equalize consumption across all agents at a given date?

(b) Does the planner redistribute resources from high wage workers to low wage workers?

Return to the individual's problem with a single non-contingent bond. Suppose now the utility function is

$$u(c_t, n_t) = \frac{c_t^{1-\gamma}}{1-\gamma} - \frac{\psi n_t^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}}$$

6. Compute an upper bound on the interest rate $\overline{r}$ such that for $r \leq \overline{r}$ the agent will always choose $a_{t+1} = 0$.

7. For $r \leq \overline{r}$ characterize conditions on parameter values such that an increase in the wage translates to an increase in hours worked.

8. Suppose that wage shocks are *iid* over time, and supppose parameters are such that for $r \leq \overline{r}$ wages and hours are negatively correlated. Will increasing r above $\overline{r}$ tend to increase or reduce the covariance between wages and hours (just provide an intuitive argument)?