MIDTERM, GRADUATE MACRO, ECON 606

Jonathan Heathcote

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Consider the following economy. There are two sectors: an apple sector, and an orange sector. Both sectors use land F and labor n to produce. The amount of land in each sector, F_o and F_a is fixed: $F_o = F_a = F$. The production technology is Cobb-Douglas:

$$Y_i = z_i F_i^{\theta} n_i^{1-\theta} \qquad i = a, o$$

where $0 \leq \theta \leq 1$. z_i, which determines sector-specific productivity, is given by

$$\begin{aligned} z_o &= 70 + (T - 70) \\ z_a &= 70 - (T - 70) \end{aligned}$$

where $T \in \{60, 61, ..., 79, 80\}$ is the average temperature in Fahrenheit in the period, and evolves over time according to a first-order Markov process defined by the transition probability matrix Π.

Infintely-lived consumers have identical preferences over a composite consumption good C and leisure l. Suppose, to start with, that labor markets are segmented: half of the population can only work on apple farms, the other half can only work on orange farms. Let superscripts denote the identity of workers / consumers: thus, for example, c_a^o denotes consumption of apples by workers who work on orange farms. Preferences for workers of type $i \in \{a, o\}$ are given by

$$E \sum_{t=0}^{\infty} \beta^t u(C_t^i, l_t^i)$$

where

$$\begin{aligned} u(C^i, l^i) &= u(C^i) + v(l^i) \\ C^i &= \left(\left(c_a^i\right)^{\frac{\sigma-1}{\sigma}} + \left(c_o^i\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma=1}} \end{aligned}$$

$\sigma \geq 0$ is the elasticity of substitution between apples and oranges.

Assume the standard inequality constraints must be respected: $c_a^i, c_o^i, l^i \geq 0$, $l^i \leq 1$.

PART 1: Consider a planner who cares equally about apple workers and orange workers.

1. Write down a recursive formulation of the planner's problem. Take care to choose appropriate state variable(s).

2. What are the planner's first order conditions?.

3. Compare the optimal consumption bundle for apple farm workers to the optimal consumption bundle for orange farm workers.

4. Consider the limiting case as $\sigma \to \infty$. Can you solve for the optimal value for the ratio $\frac{n^o}{n^a}$ as a function of the state variable(s)? If you can, do so; if not, explain what extra information would be required.

   PART 2: Now consider a competitive equilibrium. Suppose the assets traded are shares in farms, and that the number of (infinitely-divisible) shares in each sector is equal to 1. Suppose farms maximize the expected present value of dividends, and that they value dividends across dates and states according to their worker's state-specific marginal rate of substitution for the good they produce.

5. Define the (two types of) consumers' problems and the firm's problem recursively. Take care in choosing appropriate state variables for the consumers problems, and for the firm's problem.

6. Define a recursive competitive equilibrium for this economy.

   Suppose that at $t = 0$ consumers working on apple farms are endowed with half the shares in both sectors: $s^a_{o,0} = s^a_{a,0} = 0.5$. Suppose that $T_0 = 70$, and suppose the matrix Π is symmetric (so that, for example, beginning at $T = 70$, any given increase in temperature has the same probability as a fall in temperature of the same magnitude). Suppose that apples and oranges are perfect substitutes. Consider three alternative scenarios:

   (a) The benchmark for which you just defined an equilibrium: stocks can be costlessly traded, but labor is not mobile between sectors (as in the planner's problem).

   (b) Transactions costs associated with stock trade are so large, that there is never any trade in stocks (ie $s^a_{o,t} = s^a_{a,t} = 0.5$ for all $t \geq 0$). Labor is not mobile between sectors

   (c) Transactions cost are large, so $s^a_{o,t} = s^a_{a,t} = 0.5$ for all t, but after observing shocks and offered wages, workers can decide where to work.

7. Under which of these three scenarios, if any, are allocations in the competitive equilibrium equal to the allocations that solve the planner's problem? Prove / explain your answers as best you can.

8. Under scenario (a), characterize equilibrium portfolio choices.