

Consumption and Labor Supply with Partial Insurance

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Introduction

- What are the facts about inequality over the life-cycle and through time?
- What sort of theory do we need to account for these facts?
- We develop a version of the life-cycle model with labor supply in which the mapping between structural parameters and moments is transparent
- all expressions in closed-form
- We learn what model ingredients are key to explaining certain facts
- Useful guide for quantitative work with heterogeneous agents and incomplete risk-sharing

Outline

1. Facts
2. Model
3. Identification
4. Estimation results
5. Sensitivity

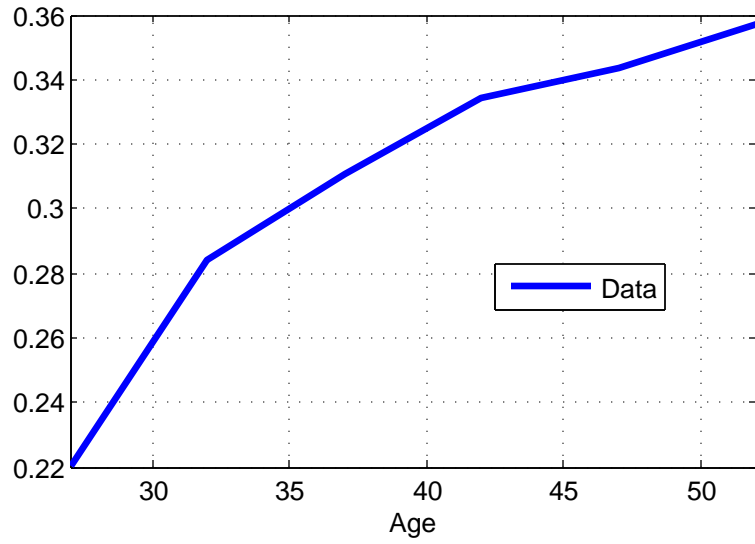
Macro Facts

- Joint evolution of within-cohort dispersion in wages, hours and consumption contains valuable information about risk and insurance when organized within the life-cycle model of consumption and labor supply ►
Storesletten, Telmer and Yaron (2004)
- Changes in cross-sectional dispersion over time contain valuable information about the nature of changes in the nature of risk over time ►
Blundell and Preston (1998), Krueger and Perri (2004)
- **Our approach:** both sets of facts are important, one would like to simultaneously account for both within life-cycle framework

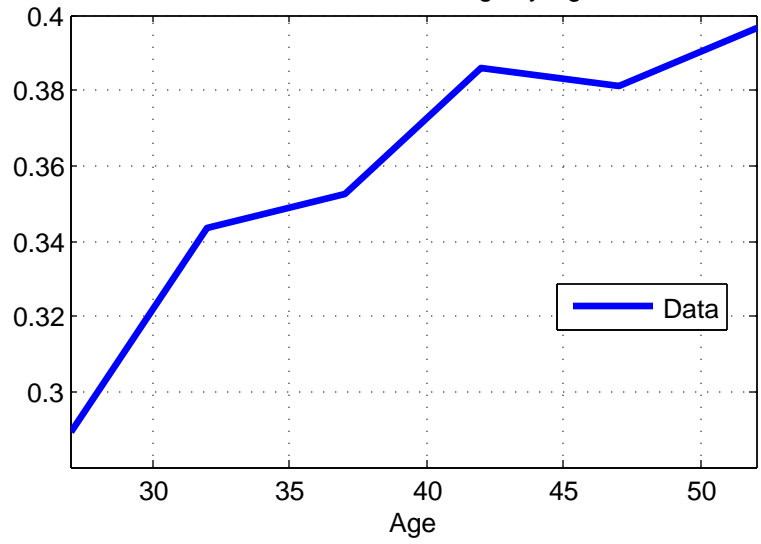
Data

- Wages, hours and earnings from PSID, 1967-1996
 - Wage computed as annual earnings divided by annual hours
 - Observations dropped if wage less than half minimum wage or if earnings top coded
- Consumption from Consumer Expenditure Survey, 1980-1996
 - Household-level data from Krueger and Perri (2004)
 - Focus on measure that excludes services from durables
- Sample Selection
 - Age range 25-54
 - Annual hours ≥ 520 , ≤ 5096
 - Same sample selection criteria applied to PSID and CEX data

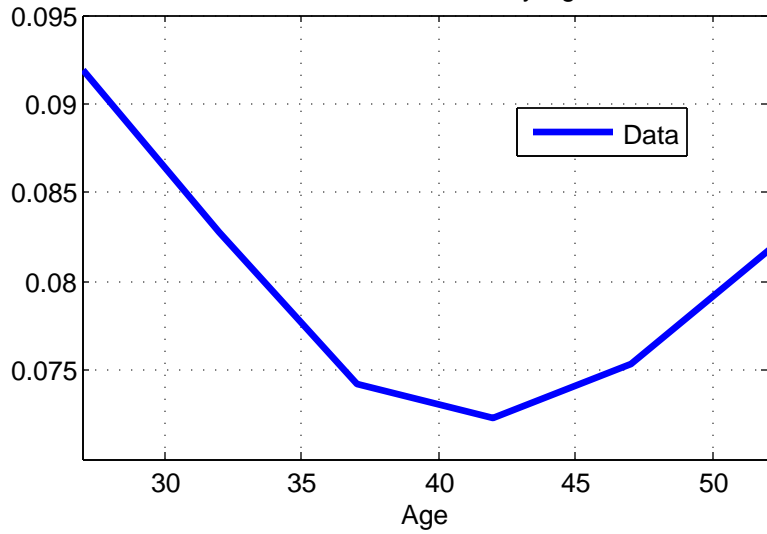
Variance of Wages by Age



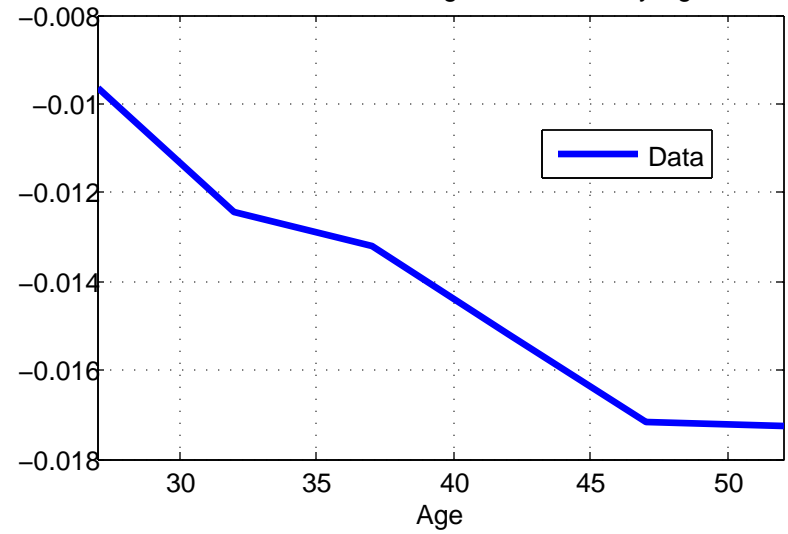
Variance of Earnings by Age



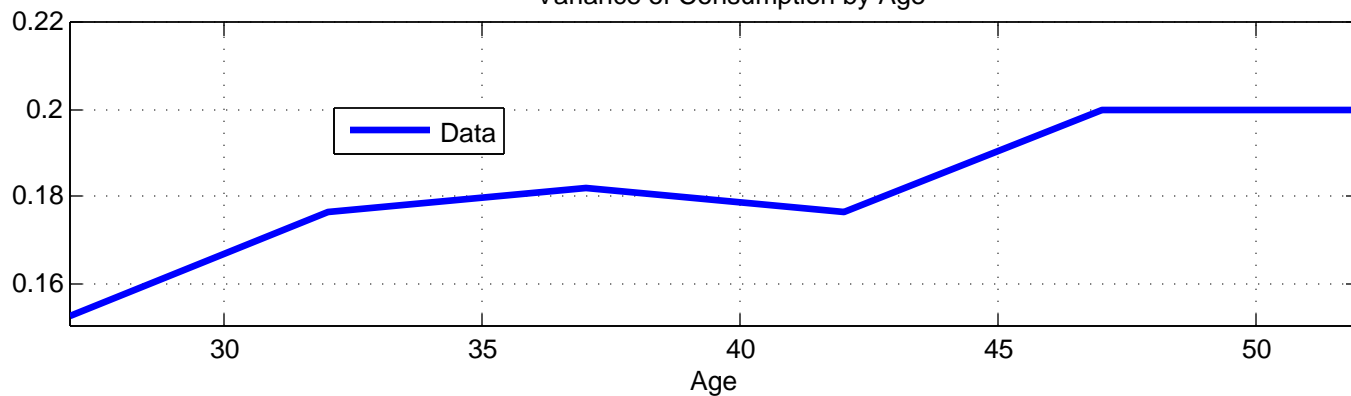
Variance of Hours by Age



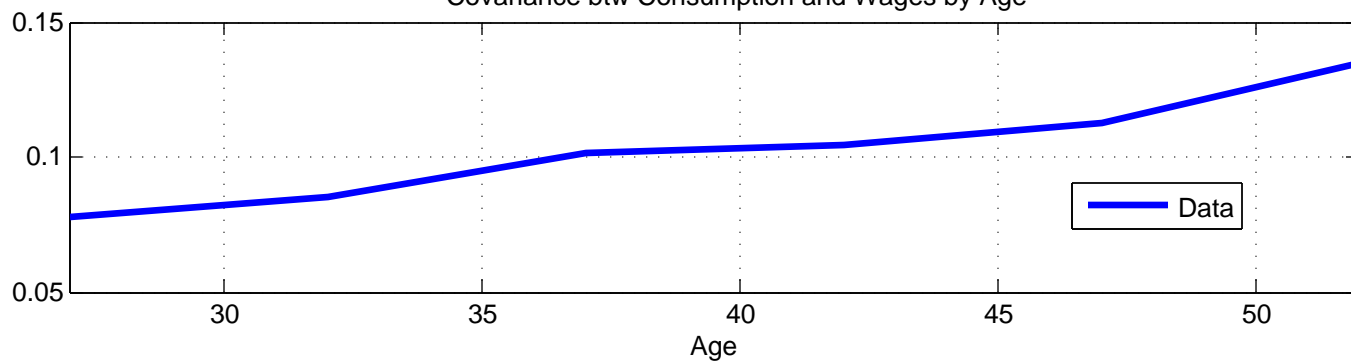
Covariance btw Wages and Hours by Age



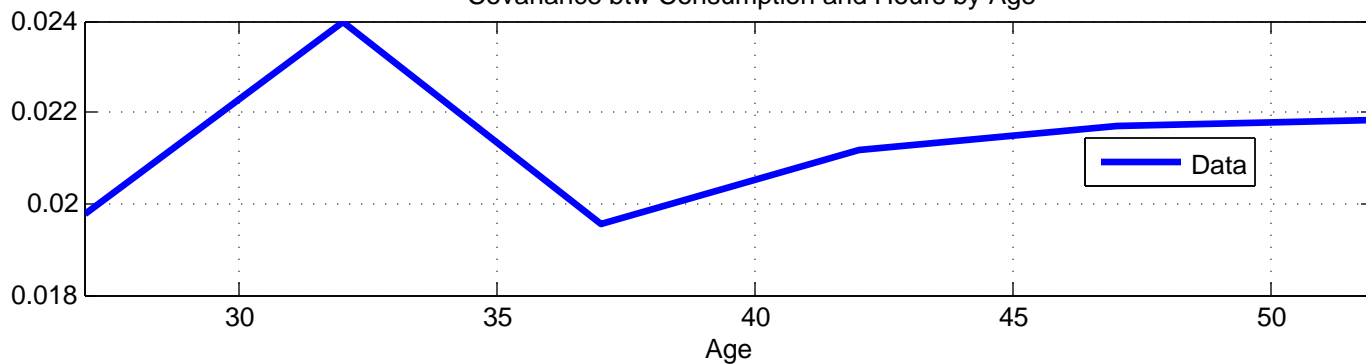
Variance of Consumption by Age



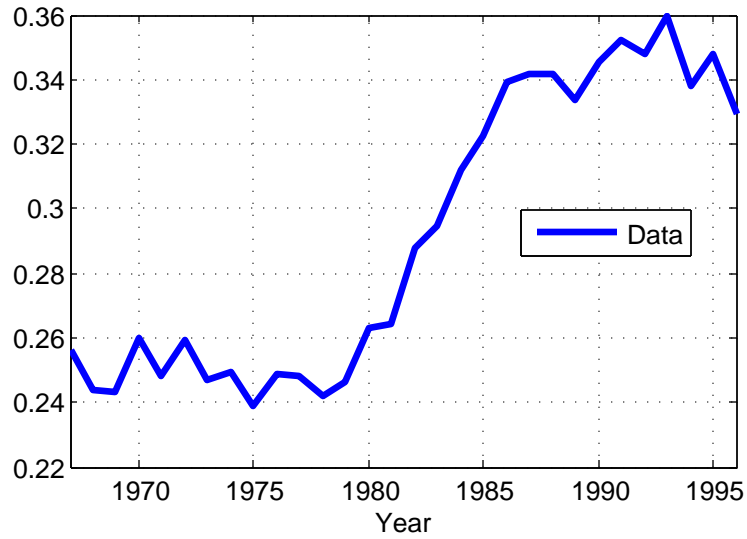
Covariance btw Consumption and Wages by Age



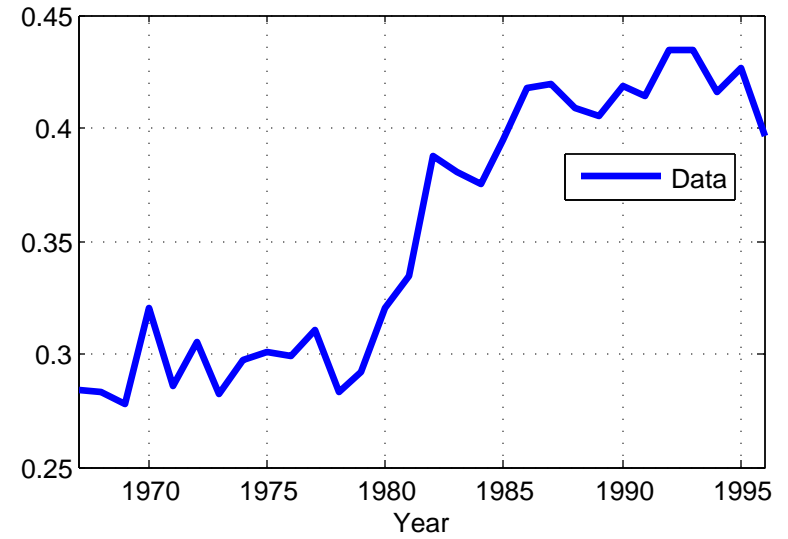
Covariance btw Consumption and Hours by Age



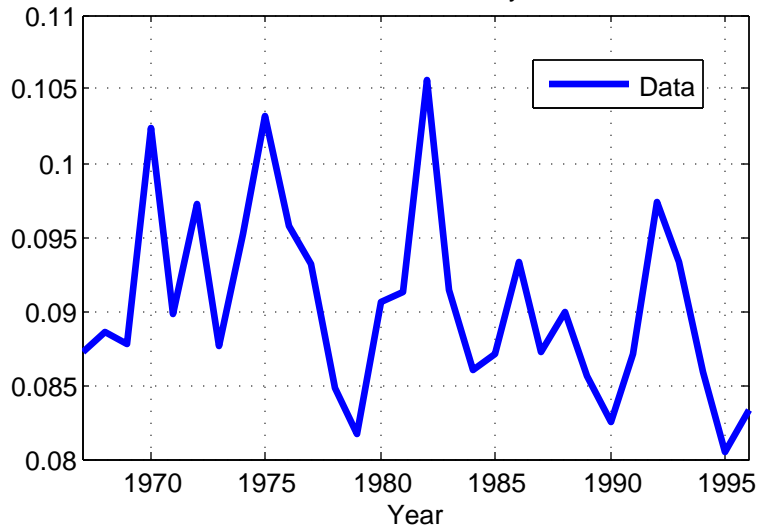
Variance of Wages by Year



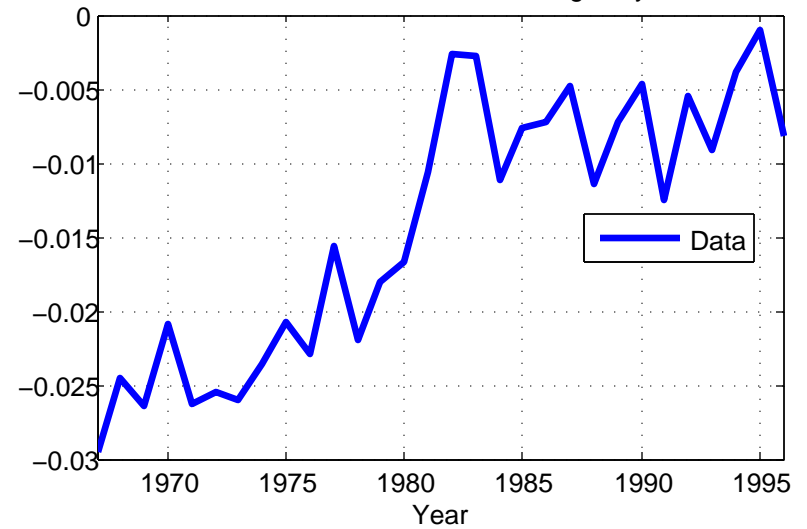
Variance of Earnings by Year



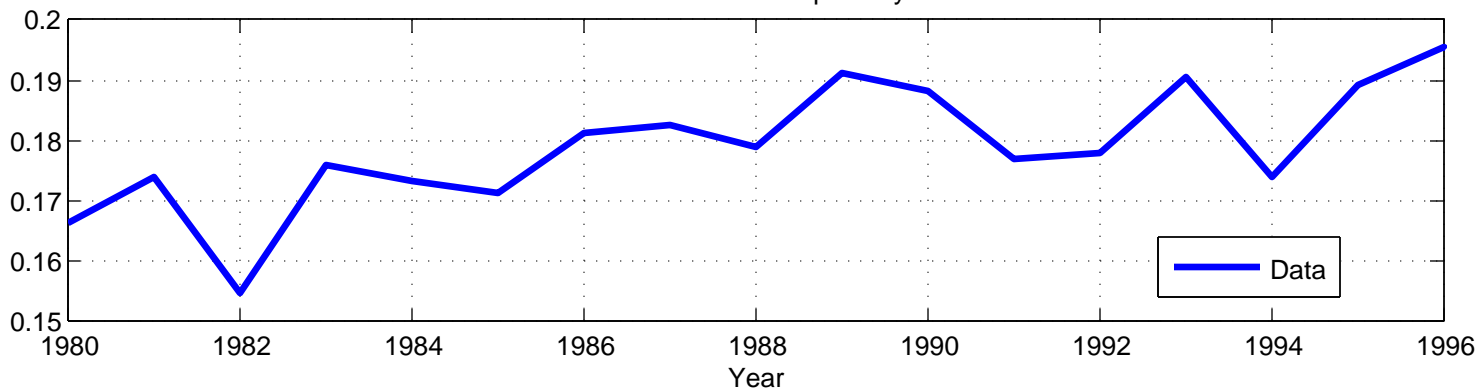
Variance of Hours by Year



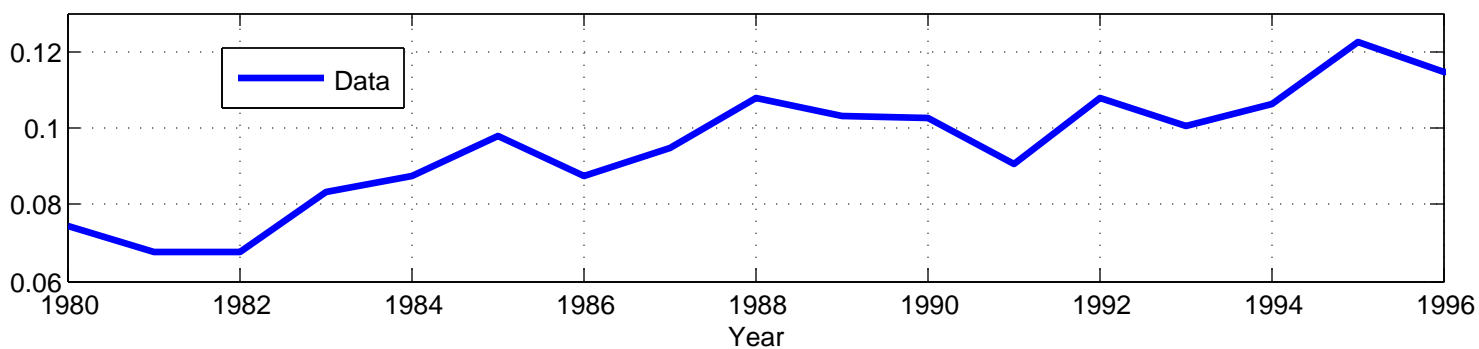
Covariance btw Hours and Wages by Year



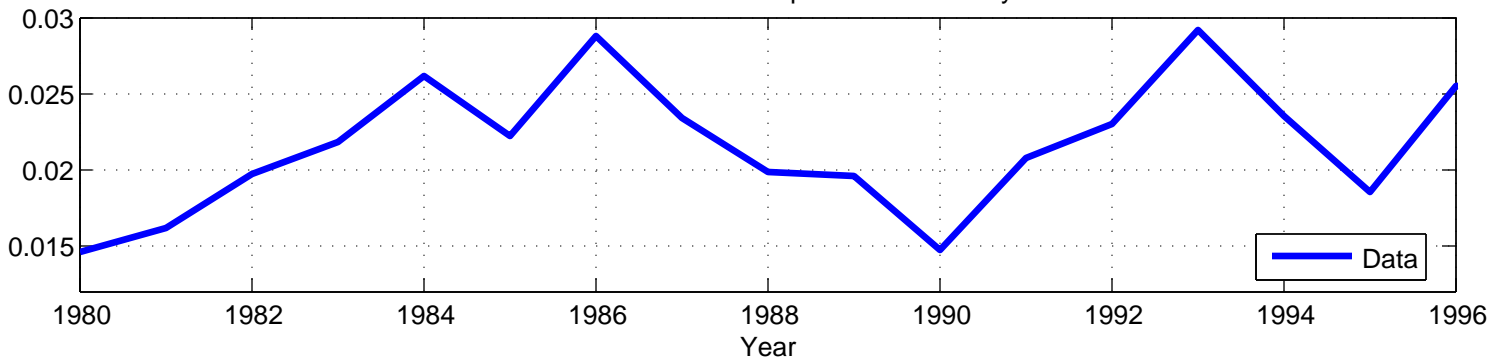
Variance of Consumption by Year



Covariance btw Wages and Consumption by Year



Covariance btw Consumption and Hours by Year



Micro Facts

- Labor economists traditionally more concerned with dynamics at the individual level: (auto) covariances of wages and hours
- **Our approach:**
 - Valuable information contained in both changes in variances (the macro picture) and the variance of changes (the micro picture)
 - Same model should be able to account for both sets of facts

Model Elements

- Perpetual youth framework with constant survival probability δ
- Agents can trade a non-contingent bond (Huggett, 1993, Aiyagari, 1994)
- In addition groups of agents potentially pool some risks
 - Groups may be families, firms, industries or countries
 - Within-group risk-sharing may be achieved by a group planner, via redistributive taxation, or through explicit insurance markets
- Agents face wage risk at the idiosyncratic, group and aggregate level
- Agents also face taste risk

Preferences

- Lifetime utility for an agent of age a born in year b is

$$\sum_{t=b}^{\infty} (\beta\delta)^{t-b} u(c_t, h_t, \zeta_t, \varphi)$$

$$u(c_t, h_t, \zeta_t, \varphi) = \frac{c_t^{1-\gamma} - 1}{1-\gamma} - \exp(\varphi + \zeta_t) \frac{h_t^{1+\sigma}}{1+\sigma}$$

- $\beta < 1$ is pure discount factor
- $\delta < 1$ is survival probability
- γ is coefficient of relative risk-aversion
- $1/\sigma$ is Frisch elasticity for labor supply
- φ and ζ_t are random individual-specific preference weights

Wages: $\log(w_t) = \alpha_t + \varepsilon_t$

- α_t : shocks that are common to group

$$\alpha_t = \alpha_{t-1} + \omega_t \quad \omega_t \sim N\left(-\frac{v_{\omega t}}{2}, v_{\omega t}\right)$$

- ε_t : idiosyncratic shocks that can be insured within-group

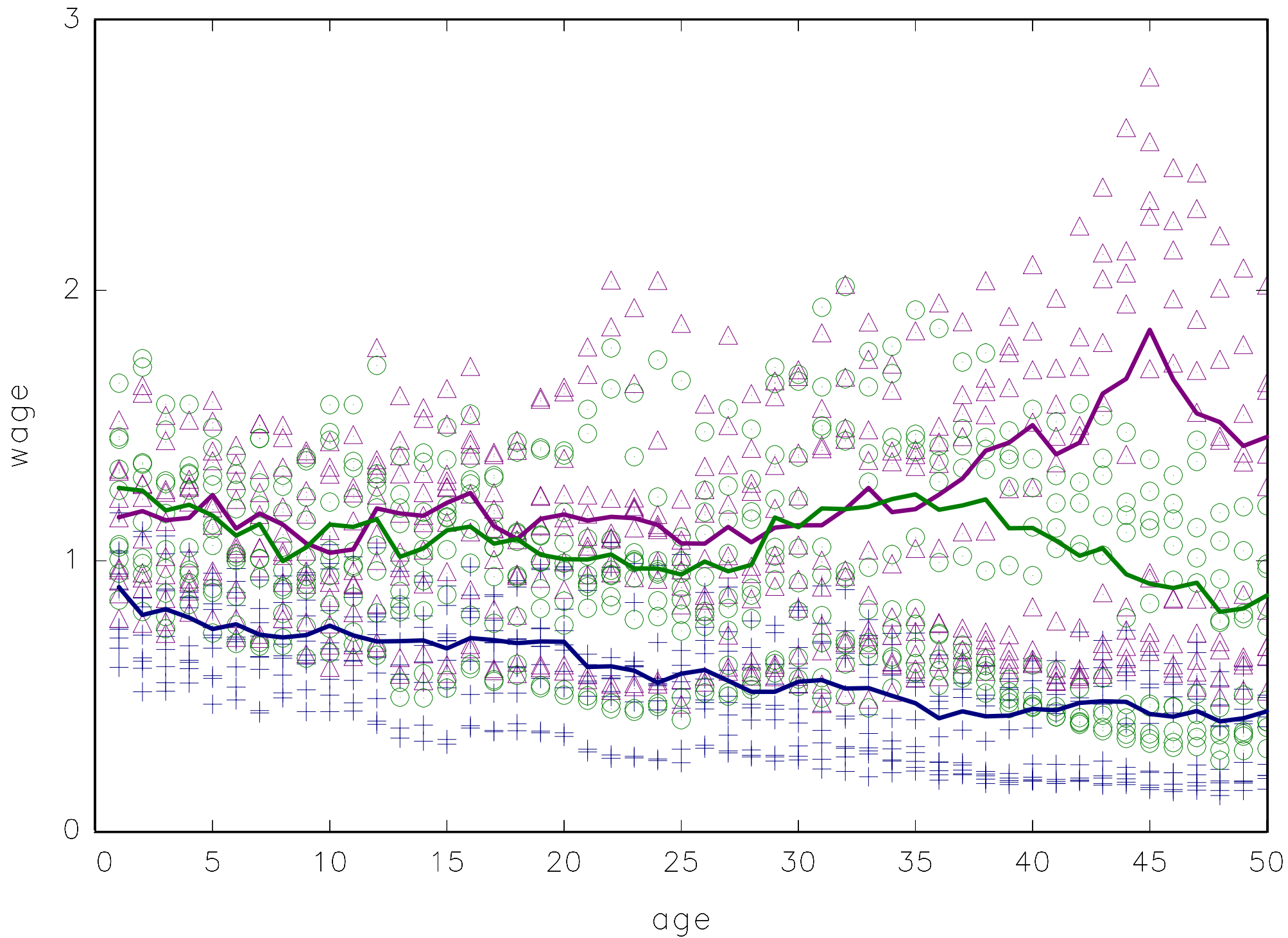
$$\varepsilon_t = \varepsilon_{t-1} + \eta_t \quad \eta_t \sim N\left(-\frac{v_{\eta t}}{2}, v_{\eta t}\right)$$

- Upon entering the labor market agents draw $\alpha_0, \varphi, \varepsilon_0$

$$\begin{pmatrix} \alpha_0 \\ \varphi \end{pmatrix} \sim N\left(\begin{matrix} -v_{\alpha 0}/2 \\ \bar{\varphi} - v_{\varphi}/2 \end{matrix}, \begin{pmatrix} v_{\alpha 0} & v_{\varphi \alpha} \\ v_{\varphi \alpha} & v_{\varphi} \end{pmatrix}\right)$$

$$\varepsilon_0 \sim N\left(-\frac{v_{\varepsilon 0}}{2}, v_{\varepsilon 0}\right) \quad \zeta_t \sim N\left(-\frac{v_{\zeta}}{2}, v_{\zeta}\right)$$

Dispersion on Three Islands



Notes

- Unconditional expected wage per efficiency unit constant and equal to 1
- Assume agents have perfect foresight over future paths for $v_{\omega t}$, $v_{\eta t}$,
 - (we can also handle uncertainty about future risk)
- All dynamics over time attributed to time effects
 - Consistent with HSV 2005
 - We will experiment with introducing cohort effects in $v_{\alpha 0}$ and $v_{\varepsilon 0}$

Market Structure

- At each date t a measure $1 - \delta$ of agents with age $a_t = 0$ is born
- Agents born onto islands indexed by φ, α_0 and $\{\omega_s\}, s = t + 1, \dots, \infty$
- At each age, agents observe all shocks, decide on hours, consumption and asset purchases
- Agents trade complete set of Arrow securities, each pays one unit of consumption for one particular combination $\lambda_{t+1} = (\omega_{t+1}, \eta_{t+1}, \zeta_{t+1})$
 - Arrow securities are only traded **within islands**
- Non-contingent bonds traded **between islands**
- Perfect annuity markets insure mortality risk in the standard fashion
- Model nests Complete Markets ($v_\varphi = v_{\alpha_0} = v_{\omega t} = 0$) and Bewley-Imrohoroglu-Huggett-Aiyagari Economy ($v_{\varepsilon 0} = v_{\eta t} = v_\zeta = 0$)

Budget Constraints

- Generic budget constraint:

$$c_t + \int Q_t(\lambda_{t+1}) B_t(\lambda_{t+1}) d\lambda_{t+1} + q_t b_t = w_t h_t + d_t$$

$$d_{t+1} = B_t(\lambda_{t+1}) + b_t$$

- Borrowing limits rule out Ponzi schemes

Definition of Equilibrium

A set of allocations $\{c_t, h_t, b_t, B_t(\lambda_{t+1})\}$ and prices $\{q_t, Q_t(\lambda_{t+1})\}$ such that:

1. Allocations maximize expected lifetime utility for the agents, taking as given initial wealth $d_t(-1)$, and prices
2. Insurance markets clear island-by-island
3. World market for non-contingent bonds clears

Equilibrium in Closed Form

- In our particular environment we can characterize allocations and prices in closed form
- Relationship between equilibrium moments and model elements / parameter values very transparent
- *Idiosyncratic risk perfectly insured*
 - Complete markets within islands
 - Can imagine a planner choosing within-island allocations
- *Island-level risk entirely uninsured*
 - Constantinides and Duffie (1996) show that when shocks are permanent it is possible to construct equilibria in which bonds are not traded
 - We extend their result by giving agents a labor choice, and by allowing sub-groups of the population to pool idiosyncratic risks

- Key assumptions for the no trade result are:
 1. Preferences in CRRA class
 2. Agents born with zero initial wealth
 3. Island-specific wage shocks permanent and multiplicative
 4. No time effects in level of α_t
 5. Within-island wage dispersion $v_{\varepsilon,t}$ grows at same rate on all islands

Allocations

Consumption and hours are given by

$$\log(c_t) = \frac{1 + \sigma}{\sigma + \gamma} \cdot \frac{v_{\varepsilon,t}(a_t) + v_{\zeta}}{2\sigma} - \frac{\varphi}{\sigma + \gamma} + \frac{1 + \sigma}{\sigma + \gamma} \cdot \alpha_t$$

$$\log(h_t) = -\frac{\gamma}{\sigma} \cdot \frac{1 + \sigma}{\sigma + \gamma} \cdot \frac{v_{\varepsilon,t}(a_t) + v_{\zeta}}{2\sigma} - \frac{\varphi}{\sigma + \gamma} + \frac{1 - \gamma}{\sigma + \gamma} \cdot \alpha_t + \frac{\varepsilon_t - \zeta_t}{\sigma}$$

where $v_{\varepsilon,t}(a_t)$ denotes variance of insurable component on islands age a at t :

$$v_{\varepsilon,t}(a_t) = v_{\varepsilon 0} + \sum_{r=t-(a_t-1)}^t v_{\eta,r}$$

Notes on Allocations

- All agents on an island get same consumption
- More productive islands eat more, lazier islands eat less
- Greater within-island dispersion in wages or preferences increases consumption
- Hours respond positively to positive idiosyncratic productivity shocks, negatively to taste shocks
 - Response proportional to Frisch (compensated) elasticity
- Direction of hours response to uninsurable wage shocks depends on γ
 - γ determines relative strength of income and substitution effects
 - Response proportional to Marshallian (uncompensated) elasticity

Prices

$$\log(q_t) = \log(\beta) - \gamma \cdot \frac{1 + \sigma}{\sigma + \gamma} \left(\frac{v_{\varepsilon,t+1}(a_t + 1) - v_{\varepsilon t}(a_t)}{2\sigma} - \left(\gamma \frac{1 + \sigma}{\sigma + \gamma} + 1 \right) \frac{v_{\omega,t+1}}{2} \right)$$

$$\log(Q_t(\lambda_{t+1})) = \log(\phi_t(\lambda_{t+1})\beta) - \gamma \cdot \frac{1 + \sigma}{\sigma + \gamma} \left(\frac{v_{\varepsilon,t+1}(a_t + 1) - v_{\varepsilon t}(a_t)}{2\sigma} + \omega_{t+1} \right)$$

- Note that $v_{\varepsilon,t+1}(a_t + 1) - v_{\varepsilon t}(a_t) = v_{\eta,t}$ which is common across islands

Macro Moments

$$\text{var}(\log w_t) = v_{\alpha,t} + v_{\varepsilon,t}$$

$$\text{var}(\log c_t) = -\frac{2(1+\sigma)}{(\sigma+\gamma)^2}v_{\varphi\alpha} + \frac{v_{\varphi}}{(\sigma+\gamma)^2} + \left(\frac{1+\sigma}{\sigma+\gamma}\right)^2 v_{\alpha,t}$$

$$\text{var}(\log h_t) = -\frac{2(1-\gamma)}{(\sigma+\gamma)^2}v_{\varphi\alpha} + \frac{v_{\varphi}}{(\sigma+\gamma)^2} + \left(\frac{1-\gamma}{\sigma+\gamma}\right)^2 v_{\alpha,t} + \frac{1}{\sigma^2}(v_{\varepsilon,t} + v_{\zeta})$$

$$\text{cov}(\log w_t, \log h_t) = -\frac{v_{\varphi\alpha}}{\sigma+\gamma} + \frac{1-\gamma}{\sigma+\gamma}v_{\alpha,t} + \frac{1}{\sigma}v_{\varepsilon,t}$$

$$\text{cov}(\log w_t, \log c_t) = -\frac{v_{\varphi\alpha}}{\sigma+\gamma} + \frac{1+\sigma}{\sigma+\gamma}v_{\alpha,t}$$

$$\text{cov}(\log h_t, \log c_t) = -\frac{(1+\sigma) + (1-\gamma)}{(\sigma+\gamma)^2}v_{\varphi\alpha} + \frac{v_{\varphi}}{(\sigma+\gamma)^2} + \frac{(1-\gamma)(1+\sigma)}{(\sigma+\gamma)^2}v_{\alpha,t}$$

Micro Moments

$$\text{var}(\Delta \log w_t) = v_{\omega t} + v_{\eta t}$$

$$\text{var}(\Delta \log c_t) = \left(\frac{1 + \sigma}{\sigma + \gamma} \right)^2 v_{\omega t}$$

$$\text{var}(\Delta \log h_t) = \left(\frac{1 - \gamma}{\sigma + \gamma} \right)^2 v_{\omega t} + \frac{1}{\sigma^2} (v_{\eta t} + 2v_{\zeta})$$

$$\text{cov}(\Delta \log w_t, \Delta \log h_t) = \frac{1 - \gamma}{\sigma + \gamma} v_{\omega t} + \frac{1}{\sigma} v_{\eta t}$$

$$\text{cov}(\Delta \log w_t, \Delta \log c_t) = \frac{1 + \sigma}{\sigma + \gamma} v_{\omega t}$$

$$\text{cov}(\Delta \log h_t, \Delta \log c_t) = \frac{(1 + \sigma)(1 - \gamma)}{(\sigma + \gamma)^2} v_{\omega t}$$

Estimation

- First stage regression on year dummies, race, gender, and a quartic in experience applied to both PSID and CEX data
- Individuals grouped into 6 non-overlapping age groups at each date: 25-29, 30-34, ...
- Minimum distance estimator (equally-weighted)
- Target moments
 - “Cross-Sectional” variances and co-variances by age and year (Macro Facts)
 - Variances and covariances of individual changes for hours and wages (Micro Facts)

Measurement Error

- Assume classical measurement error in all variables
- Recognize the wages inherit measurement error in earnings and hours:
$$w_t = \alpha_t + \varepsilon_t + \mu_{yt} - \mu_{ht}$$
- Assume the distributions for measurement error are constant over time
- Estimate these variances along with structural model parameters

Proof of Identification

1. Changes in (co)variances identify $v_{\omega t}$, $v_{\eta t}$, γ and σ :

$$\Delta var(\log c_t) = \left(\frac{1 + \sigma}{\sigma + \gamma} \right)^2 \Delta v_{\alpha, t}$$

$$\Delta var(\log h_t) = \left(\frac{1 - \gamma}{\sigma + \gamma} \right)^2 \Delta v_{\alpha, t} + \frac{1}{\sigma^2} \Delta v_{\varepsilon, t}$$

$$\Delta var(\log w_t) = \Delta v_{\alpha, t} + \Delta v_{\varepsilon, t}$$

$$\Delta cov(\log w_t, \log h_t) = \frac{(1 - \gamma)(1 + \sigma)}{(\sigma + \gamma)^2} \Delta v_{\alpha, t} + \frac{1 + \sigma}{\sigma^2} \Delta v_{\varepsilon, t}$$

2. Sequences of cross-sectional data identify v_{ζ} and $v_{\mu y}$, $v_{\mu h}$:

$$var(\Delta \log w_t) - \Delta var(\log w_t) = 2(v_{\mu y} + v_{\mu h})$$

$$var(\Delta \log h_t) - \Delta var(\log h_t) = 2\left(\frac{v_{\zeta}}{\sigma^2} + v_{\mu h}\right)$$

$$cov(\Delta \log w_t, \Delta \log h_t) - \Delta cov(\log w_t, \log h_t) = -2v_{\mu h}$$

3. Levels of dispersion identify $v_{\alpha 0}$, $v_{\varepsilon 0}$, v_{φ} and $v_{\alpha\varphi}$

$$\text{var}(\log w_t) = v_{\alpha,t} + v_{\varepsilon,t} + v_{\mu y} + v_{\mu h}$$

$$\text{var}(\log h_t) = -\frac{2(1-\gamma)}{(\sigma+\gamma)^2}v_{\varphi\alpha} + \frac{v_{\varphi}}{(\sigma+\gamma)^2} + \left(\frac{1-\gamma}{\sigma+\gamma}\right)^2 v_{\alpha,t} + \frac{v_{\varepsilon,t}}{\sigma^2} + \frac{v_{\zeta}}{\sigma^2} + v_{\mu h}$$

$$\text{cov}(\log w_t, \log h_t) = -\frac{v_{\varphi\alpha}}{\sigma+\gamma} + \frac{1-\gamma}{\sigma+\gamma}v_{\alpha,t} + \frac{1}{\sigma}v_{\varepsilon,t} - v_{\mu h}$$

$$\text{cov}(\log c_t, \log h_t) = -\frac{(1+\sigma) + (1-\gamma)}{(\sigma+\gamma)^2}v_{\varphi\alpha} + \frac{v_{\varphi}}{(\sigma+\gamma)^2} + \frac{(1-\gamma)(1+\sigma)}{(\sigma+\gamma)^2}v_{\alpha,t}$$

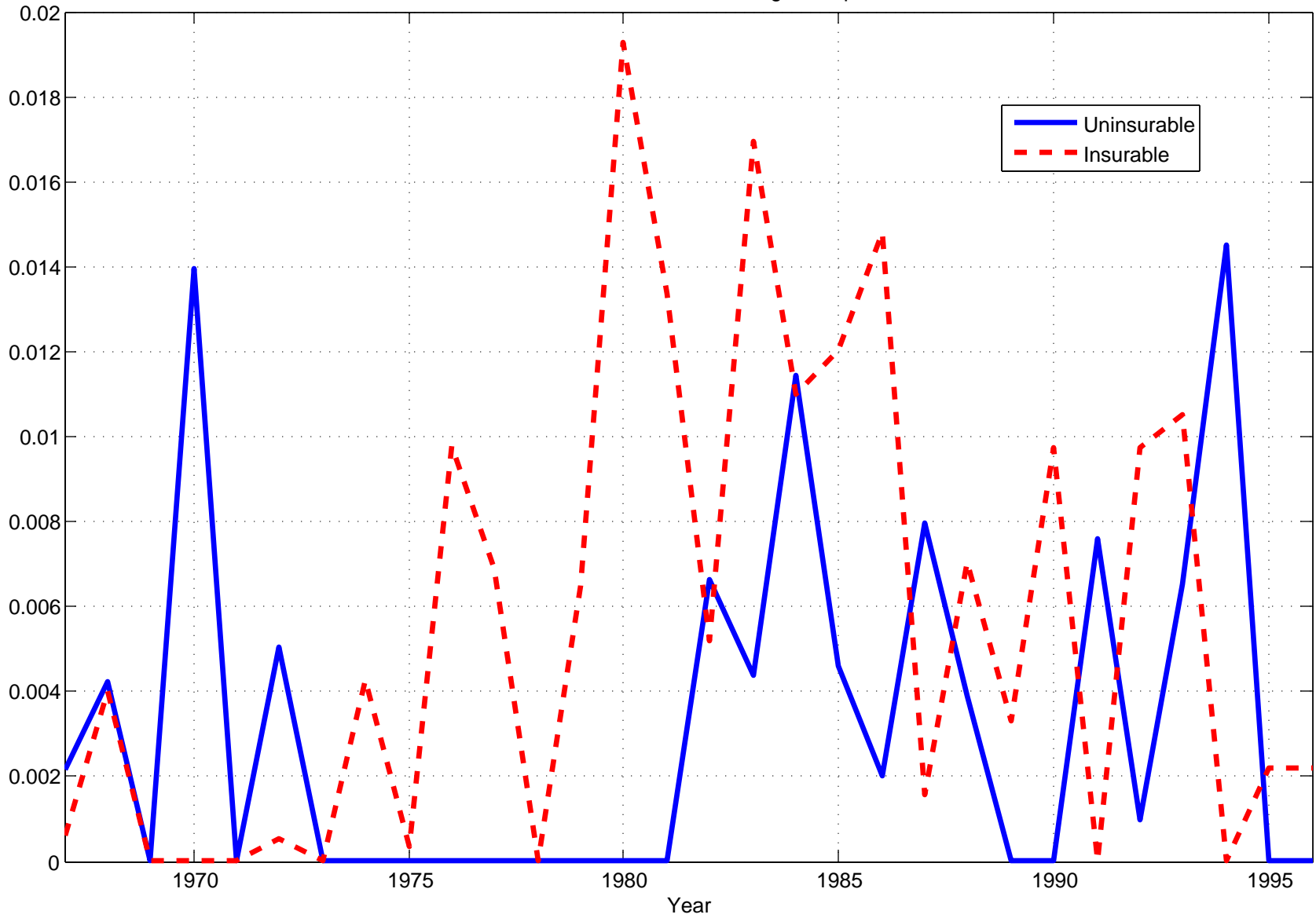
4. Expression for $\text{var}(\log c_t)$ identifies $v_{\mu c}$

$$\text{var}(\log c_t) = -\frac{2(1+\sigma)}{(\sigma+\gamma)^2}v_{\varphi\alpha} + \frac{v_{\varphi}}{(\sigma+\gamma)^2} + \left(\frac{1+\sigma}{\sigma+\gamma}\right)^2 v_{\alpha,t} + v_{\mu c}$$

Parameter Estimates

γ	σ	$v_{\alpha 0}$	$v_{\varepsilon 0}$	v_{η}	v_{ω}
2.30 (0.041)	6.94 (0.008)	0.079 (0.002)	0.069 (0.002)	0.0058 (0.0059)	0.0031 (0.0058)
v_{φ}	$v_{\alpha\varphi}$	v_{ζ}	$v_{\mu c}$	$v_{\mu y}$	$v_{\mu h}$
3.22 (0.016)	0.014 (0.006)	1.20 (0.001)	0.059 (0.0001)	0.045 (0.0000)	0.016 (0.0000)

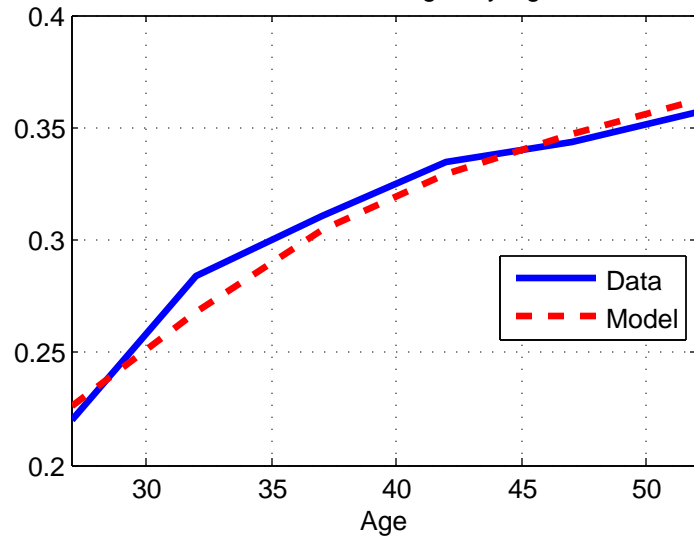
Variance of Innovations to Wage Components



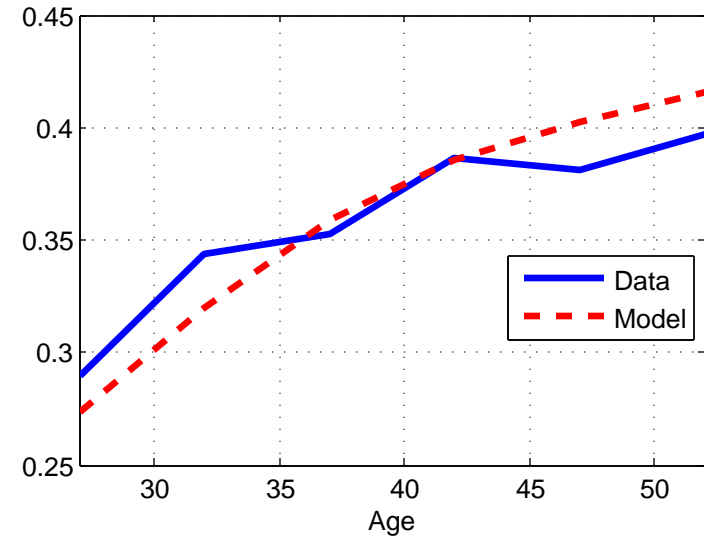
Estimates Discussion

- Estimates for γ and σ pretty standard:
 - Implied Frisch elasticity is 0.14 - 0.16
 - MaCurdy (1981) baseline estimate 0.15
- Estimates for measurement error
 - Cogley (2002) uses $var(\Delta \log c)$ to estimate $v_{\mu c}$
 - In our model $2v_{\mu c} = var(\Delta \log c) - \Delta var(\log c)$
 - Cogley's estimate for $var(\Delta \log c)$ implies $v_{\mu c} = 0.07$ in our economy, c.f. our estimate of 0.06
 - French (2002), using PSID validation study, estimates $v_{\mu h} = 0.017$, c.f. our estimate of 0.016
 - Bound et. al (1994) find 22% of variance of individual earnings growth in PSID is measurement error; our estimates imply 60%

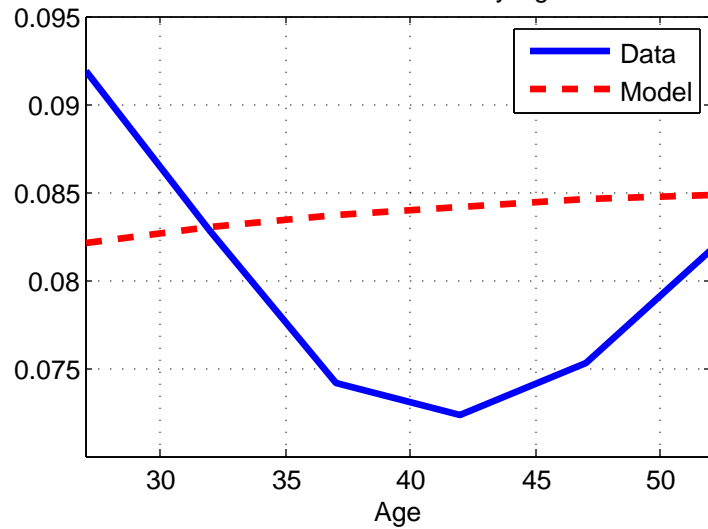
Variance of Wages by Age



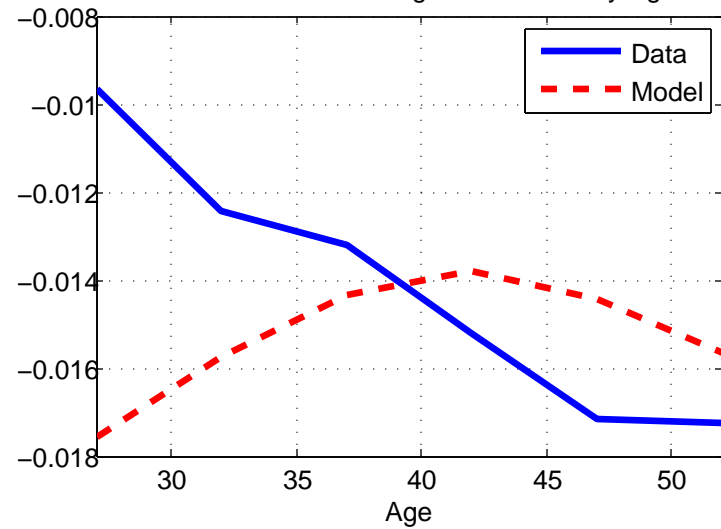
Variance of Earnings by Age

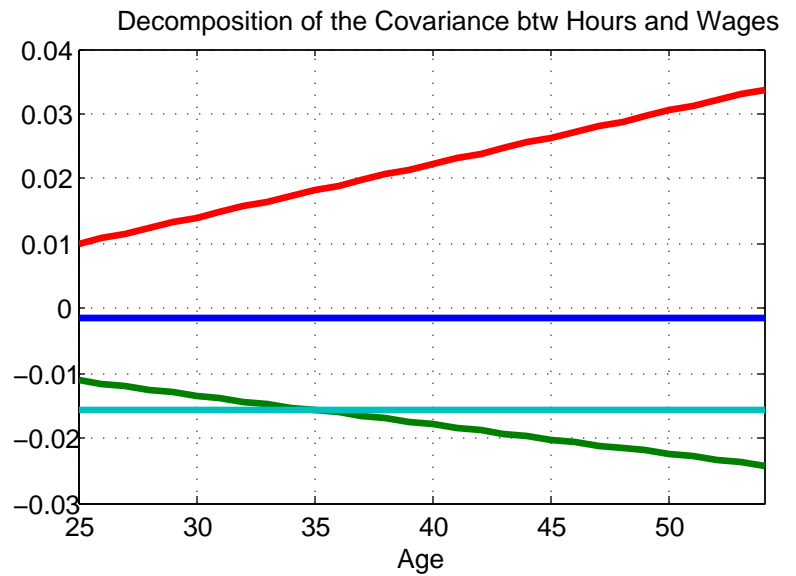
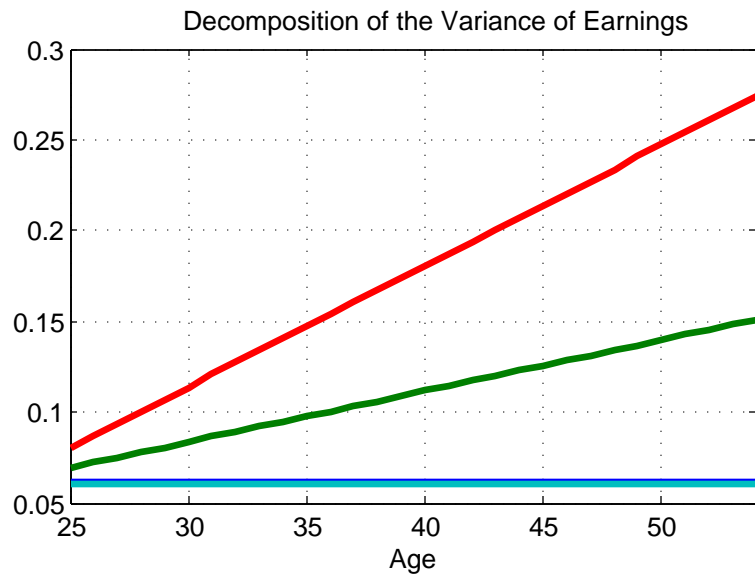
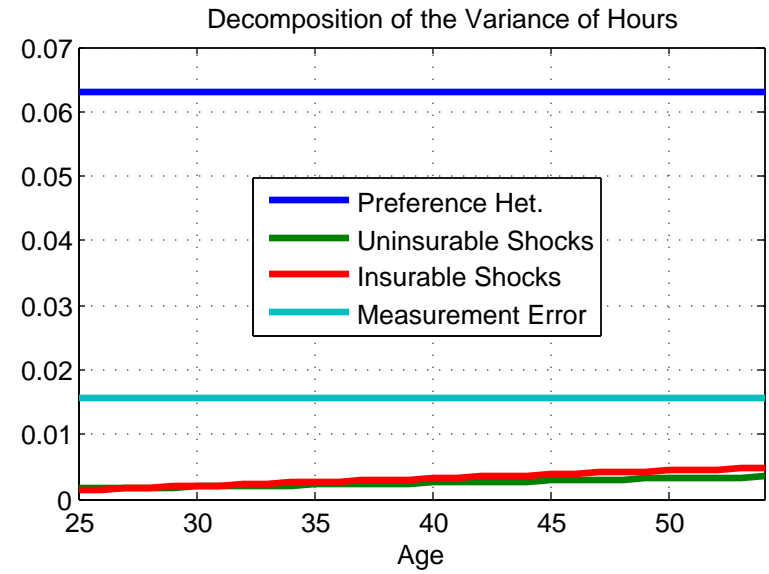
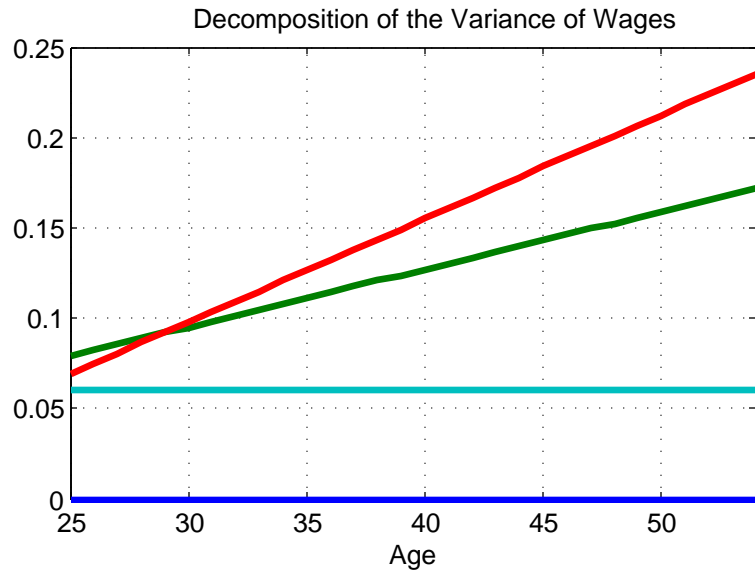


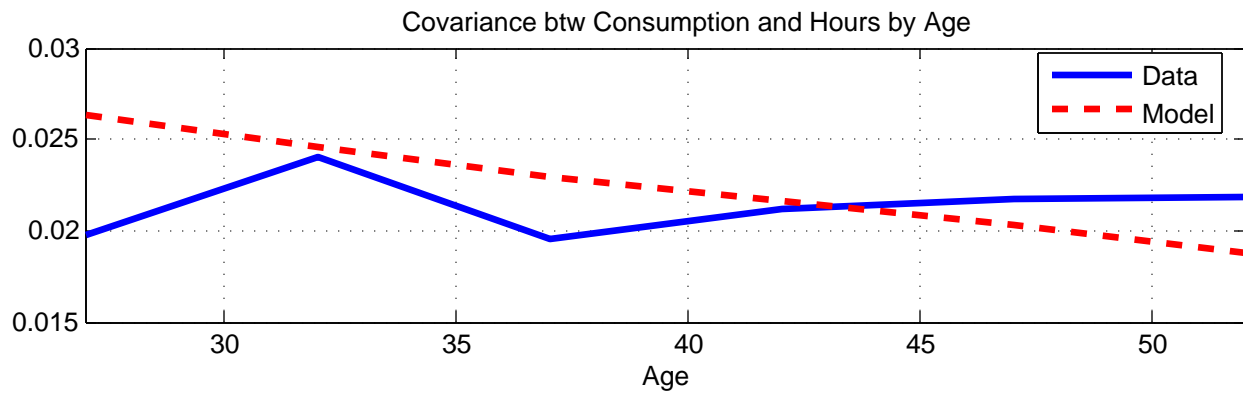
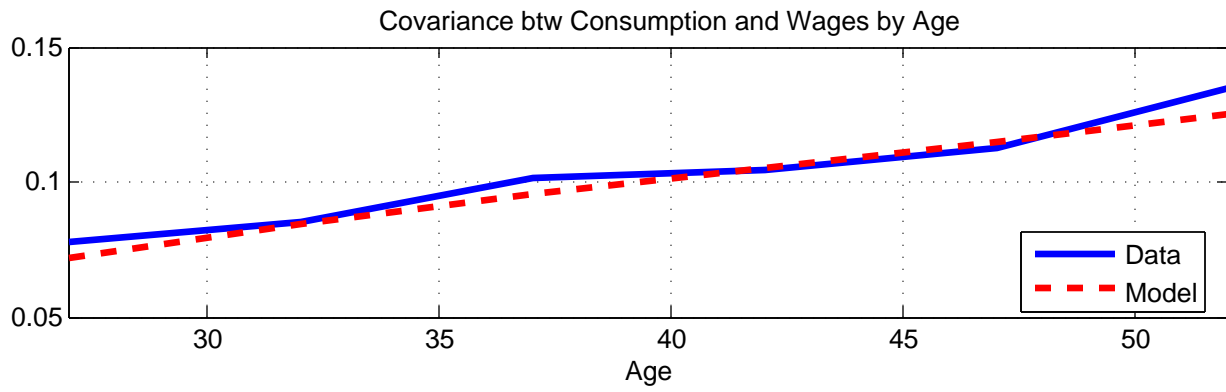
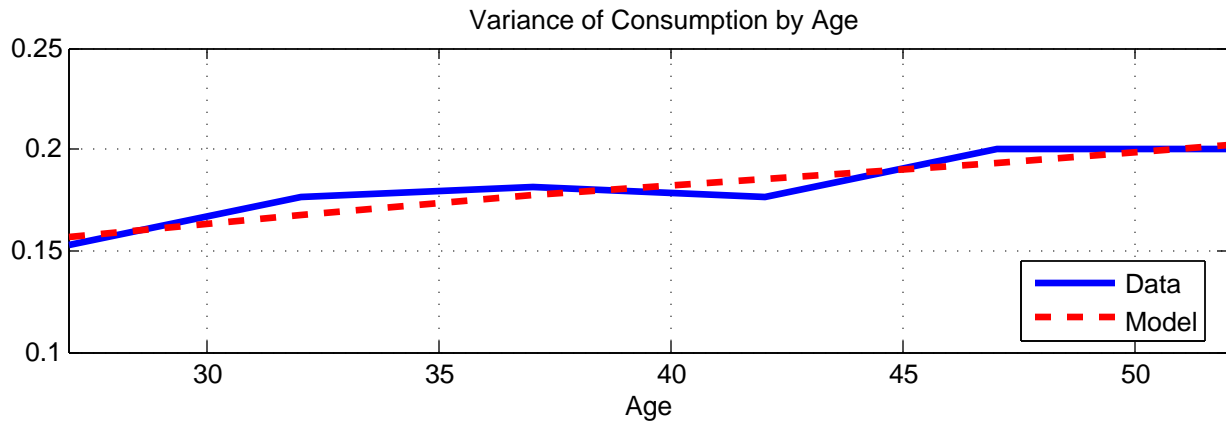
Variance of Hours by Age



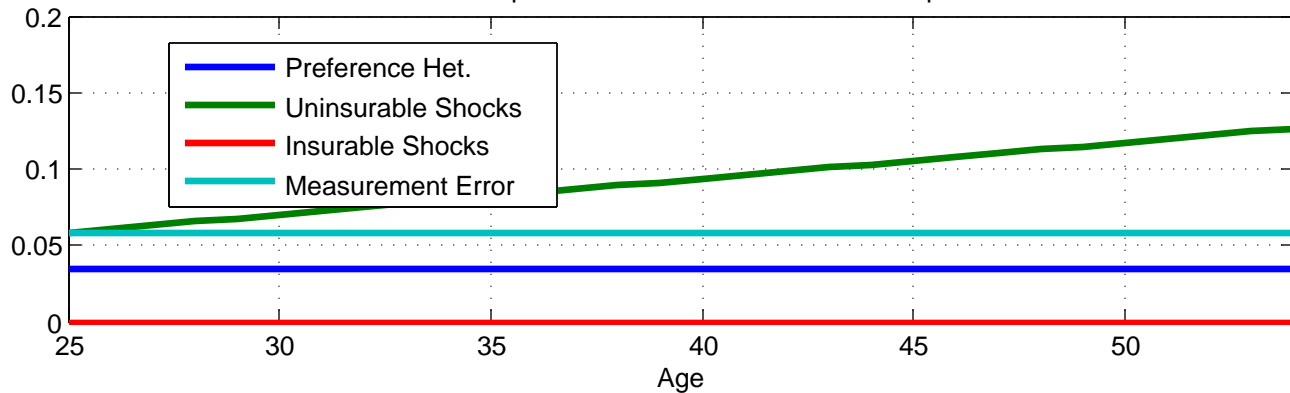
Covariance btw Wages and Hours by Age



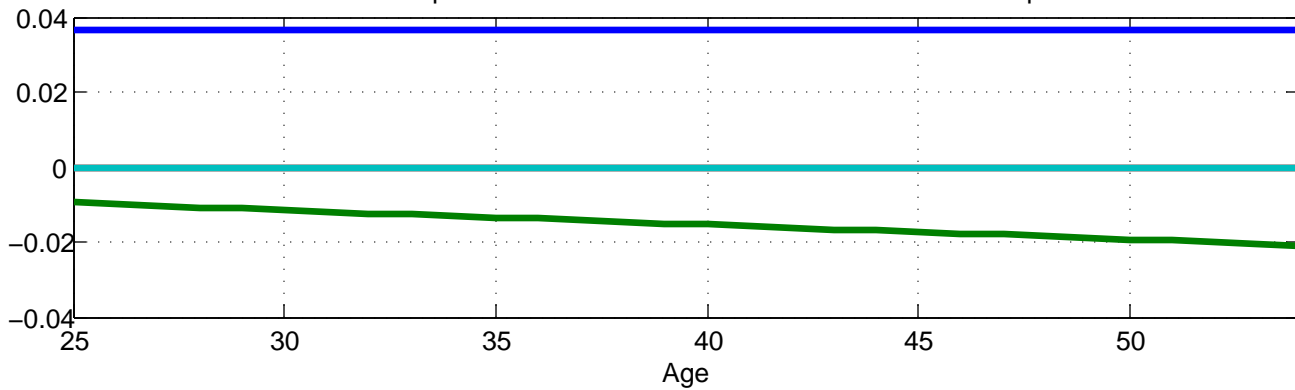




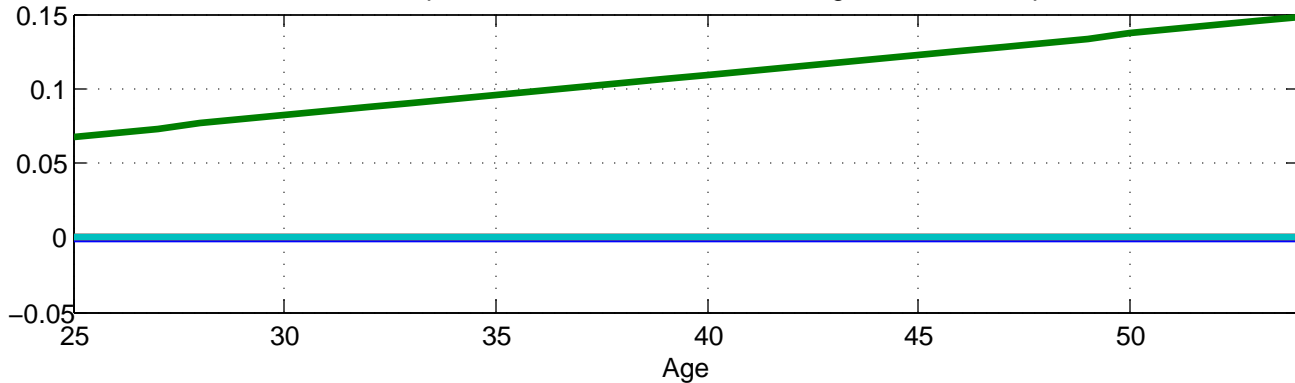
Decomposition of the Variance of Consumption



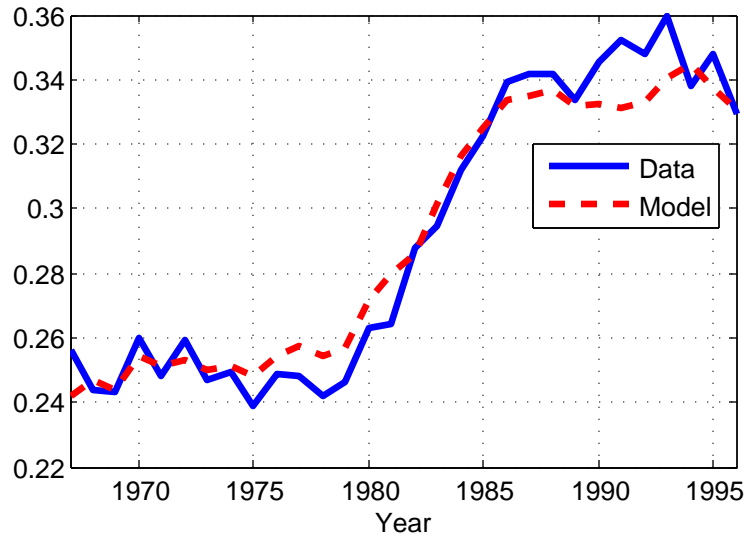
Decomposition of the Covariance btw Hours and Consumption



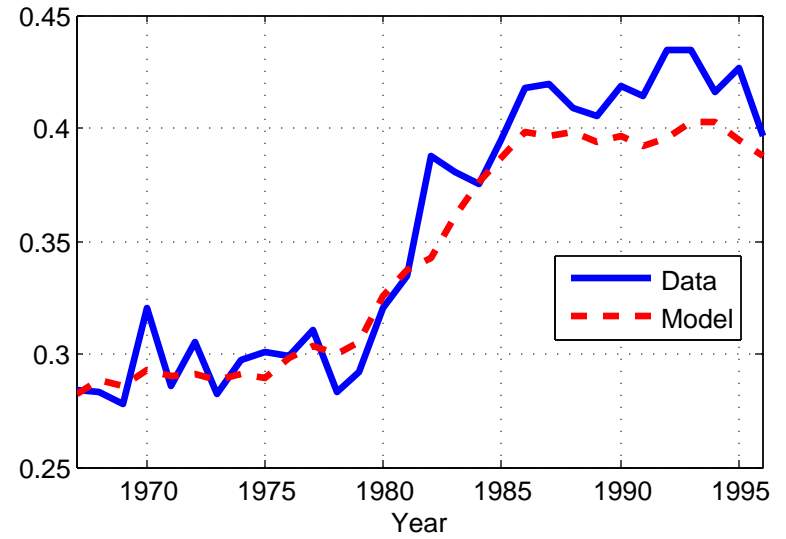
Decomposition of the Covariance btw Wages and Consumption



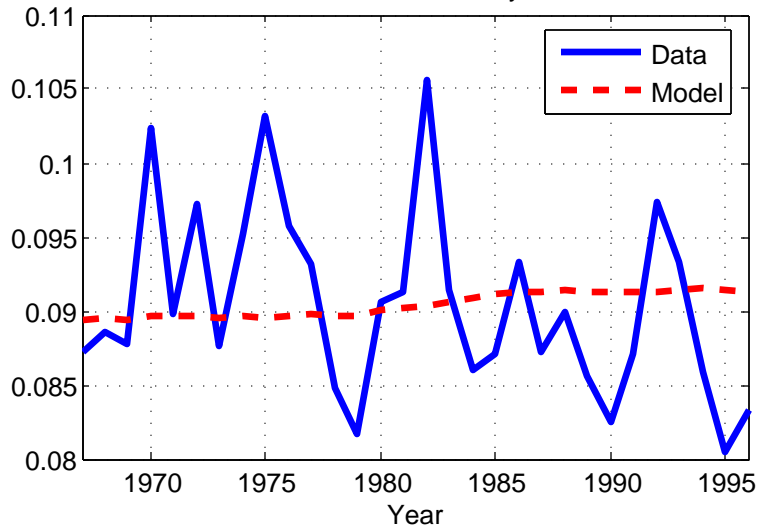
Variance of Wages by Year



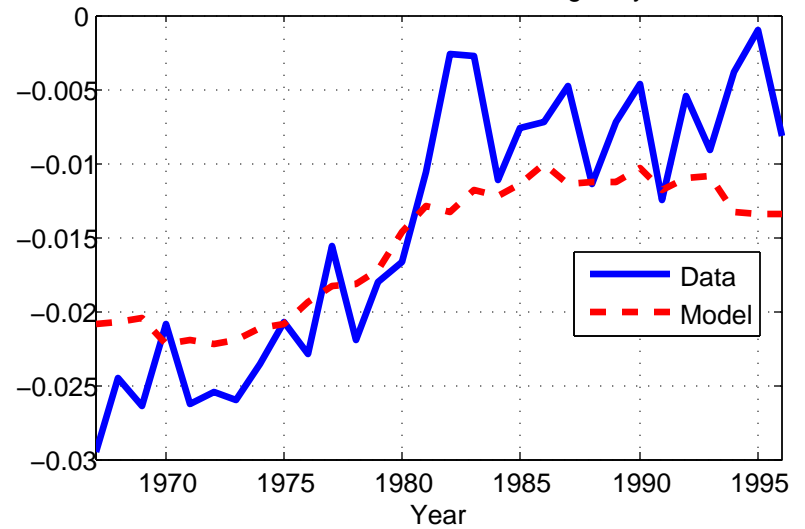
Variance of Earnings by Year



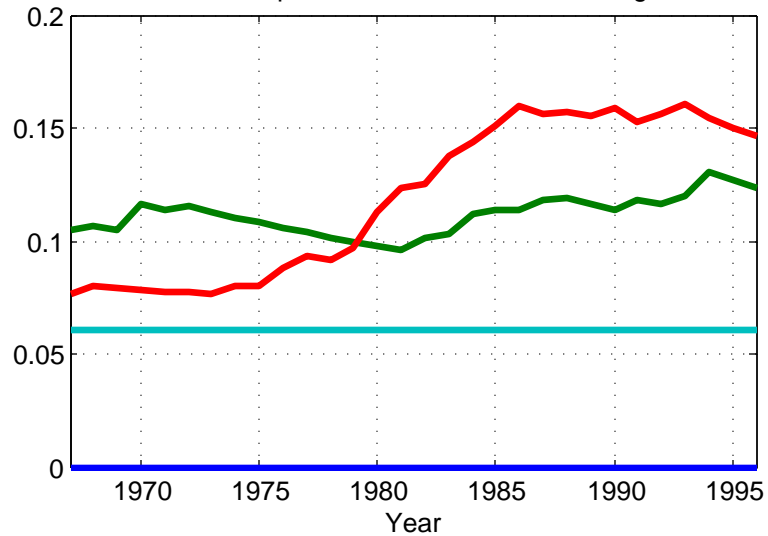
Variance of Hours by Year



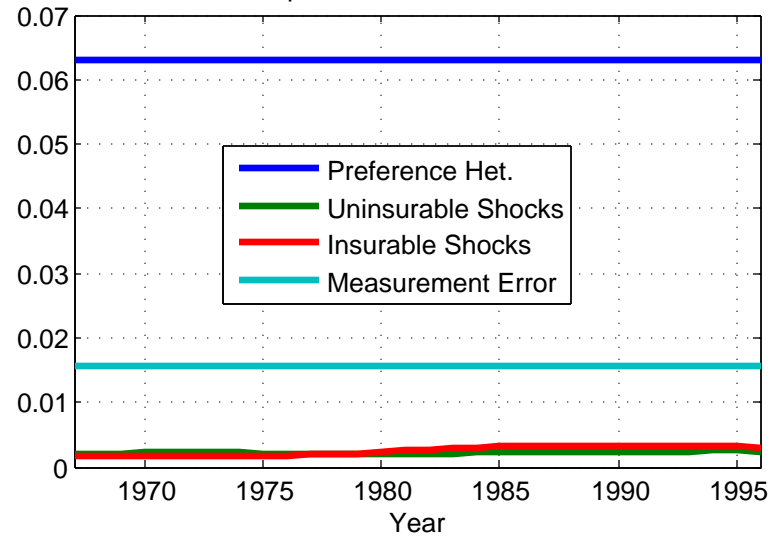
Covariance btw Hours and Wages by Year



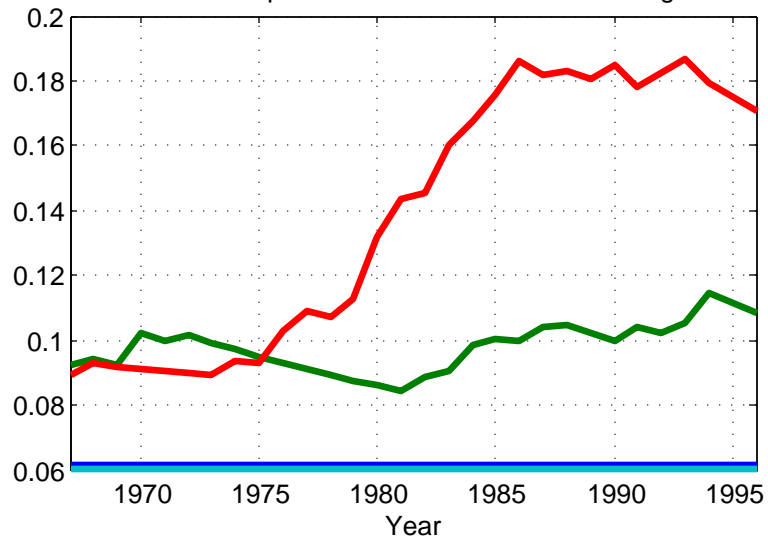
Decomposition of the Variance of Wages



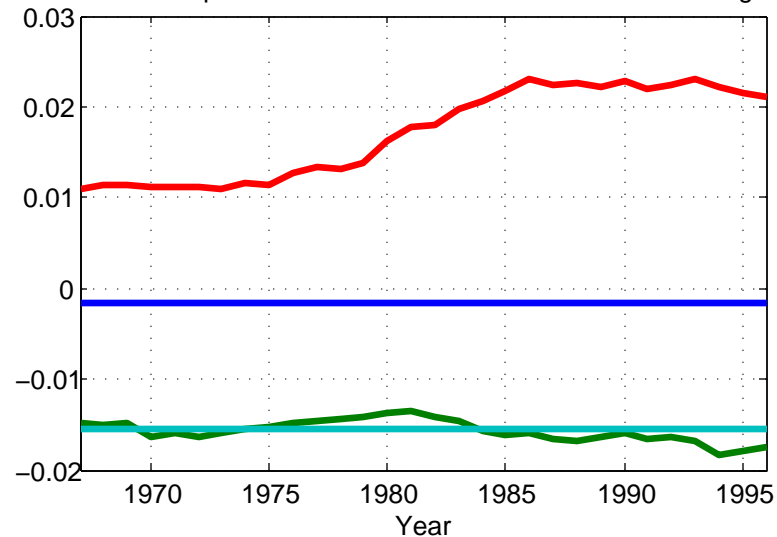
Decomposition of the Variance of Hours



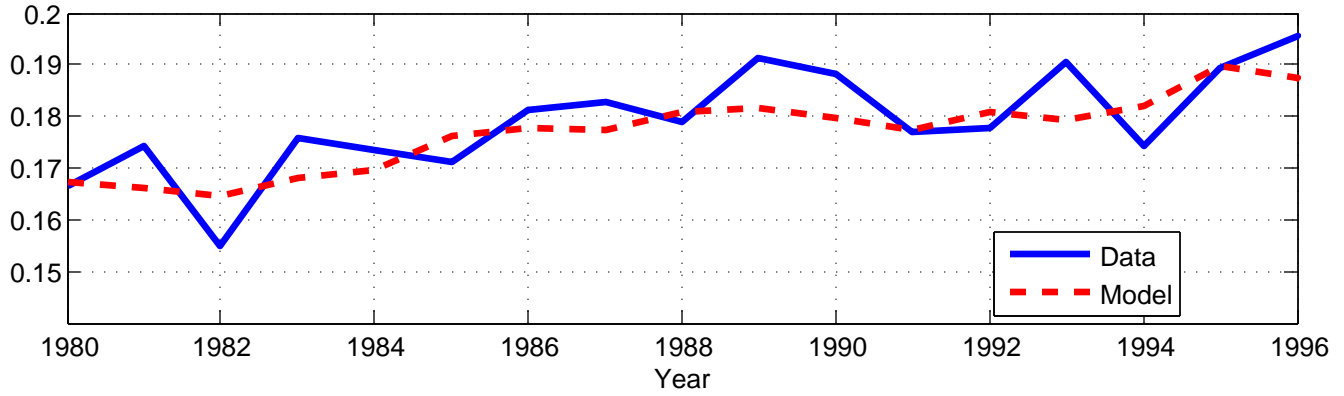
Decomposition of the Variance of Earnings



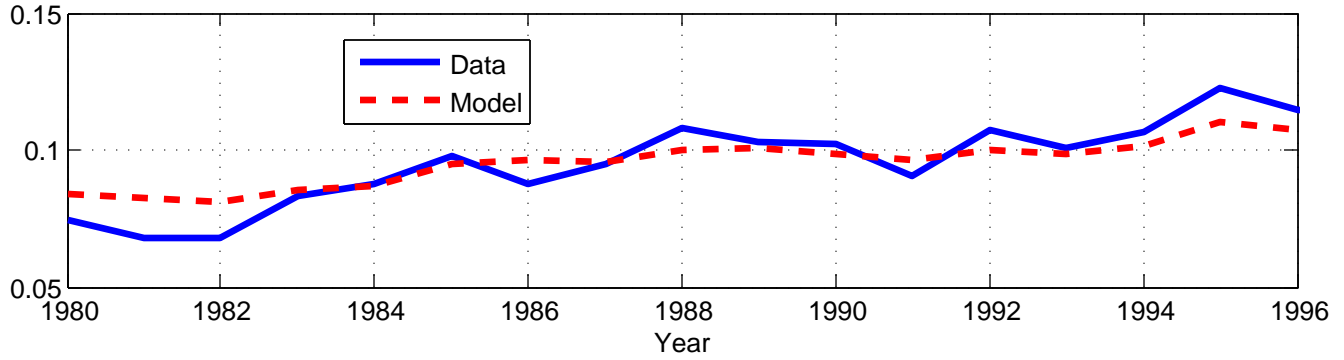
Decomposition of the Covariance btw Hours and Wages



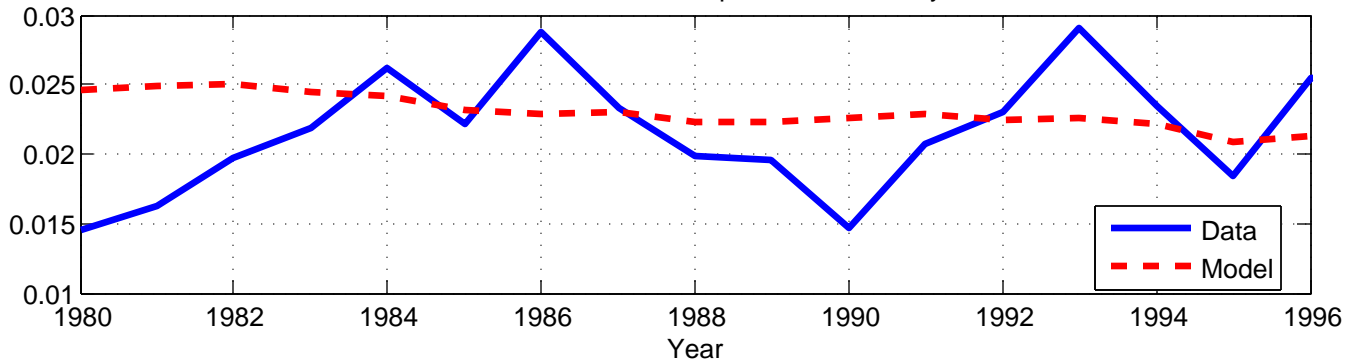
Variance of Consumption by Year



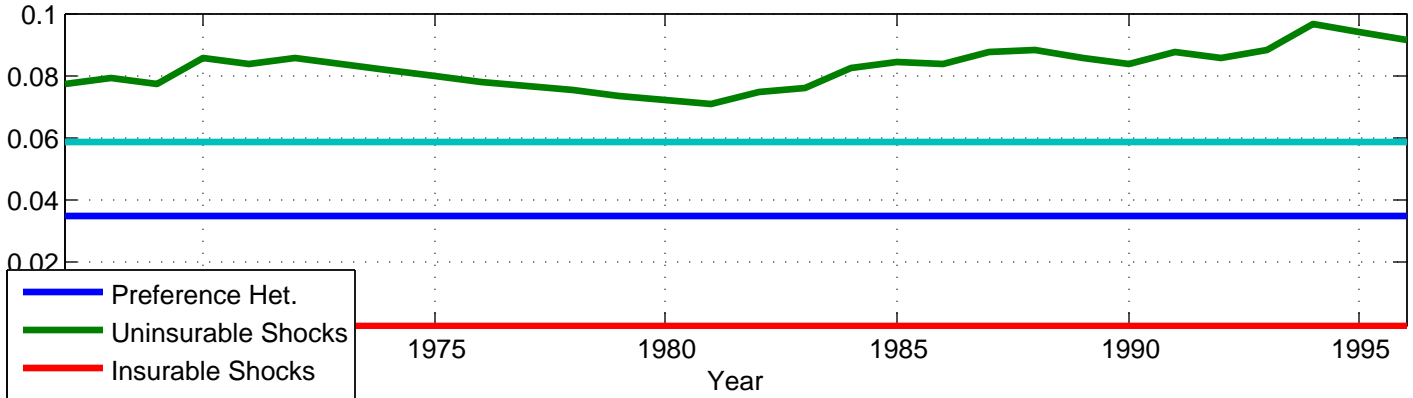
Covariance btw Wages and Consumption by Year



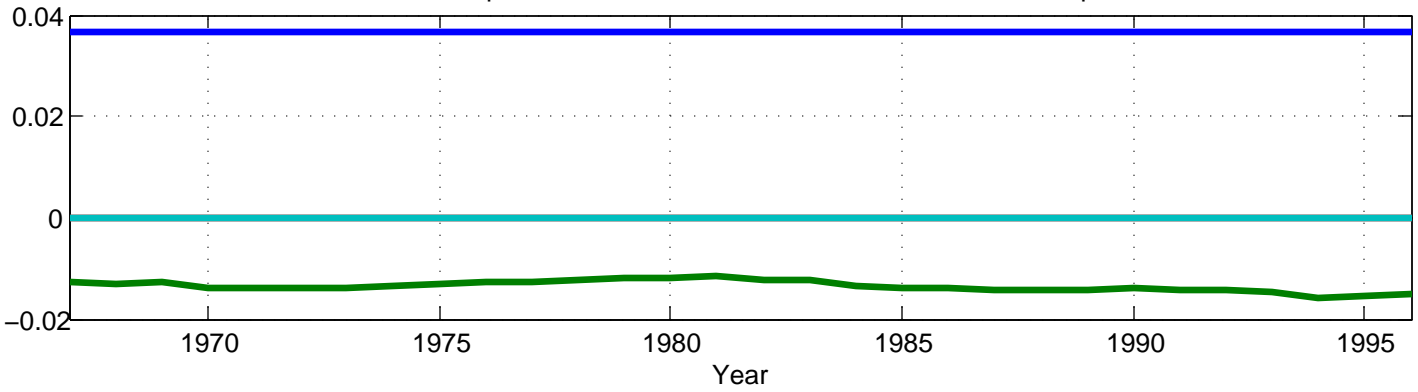
Covariance btw Consumption and Hours by Year



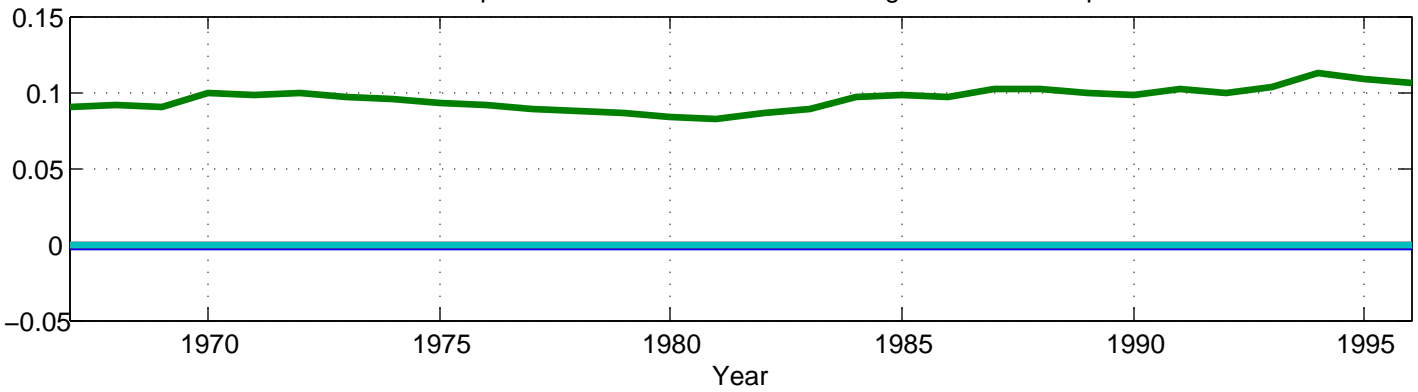
Decomposition of the Variance of Consumption



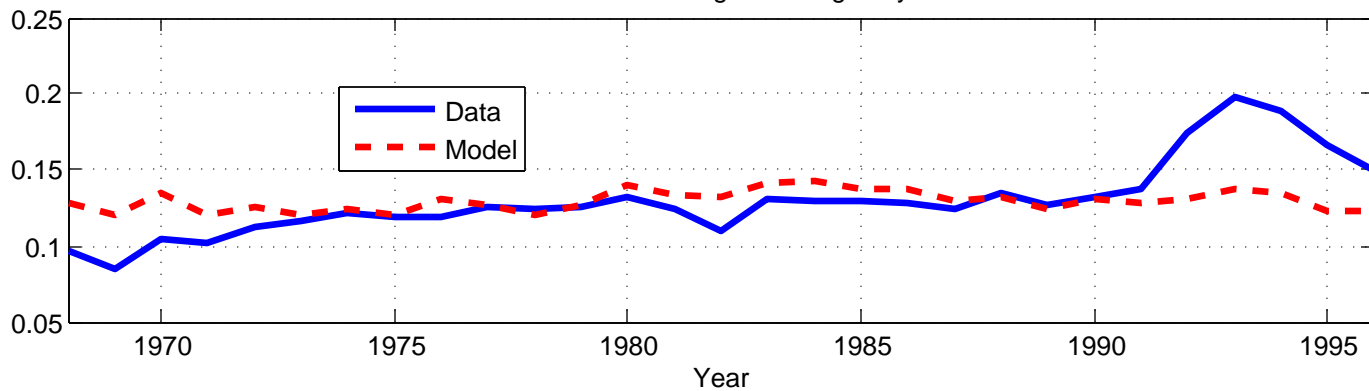
Decomposition of the Covariance btw Hours and Consumption



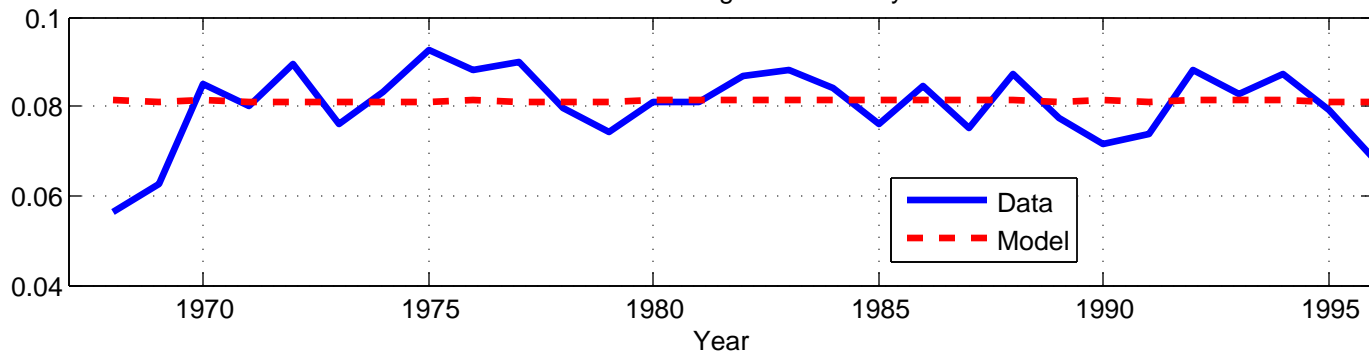
Decomposition of the Covariance btw Wages and Consumption



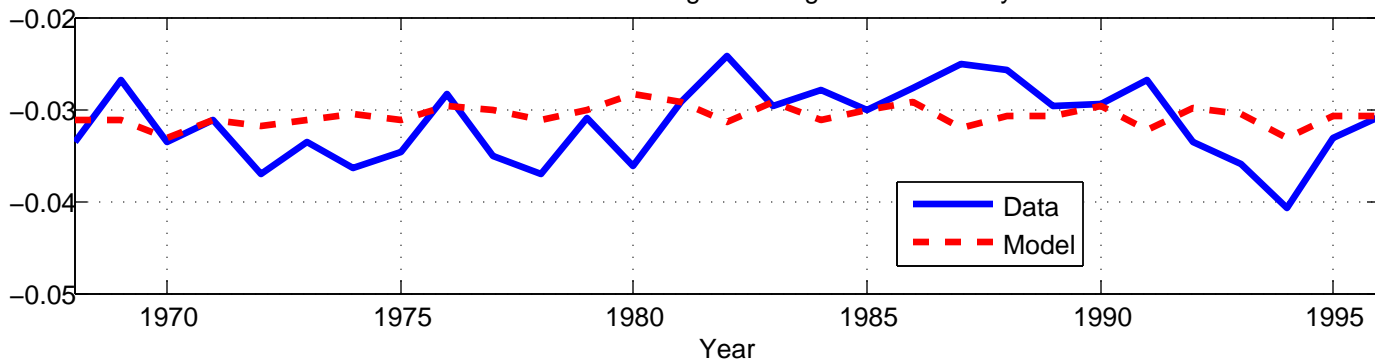
Variance of Changes in Wages by Year



Variance of Changes in Hours by Year



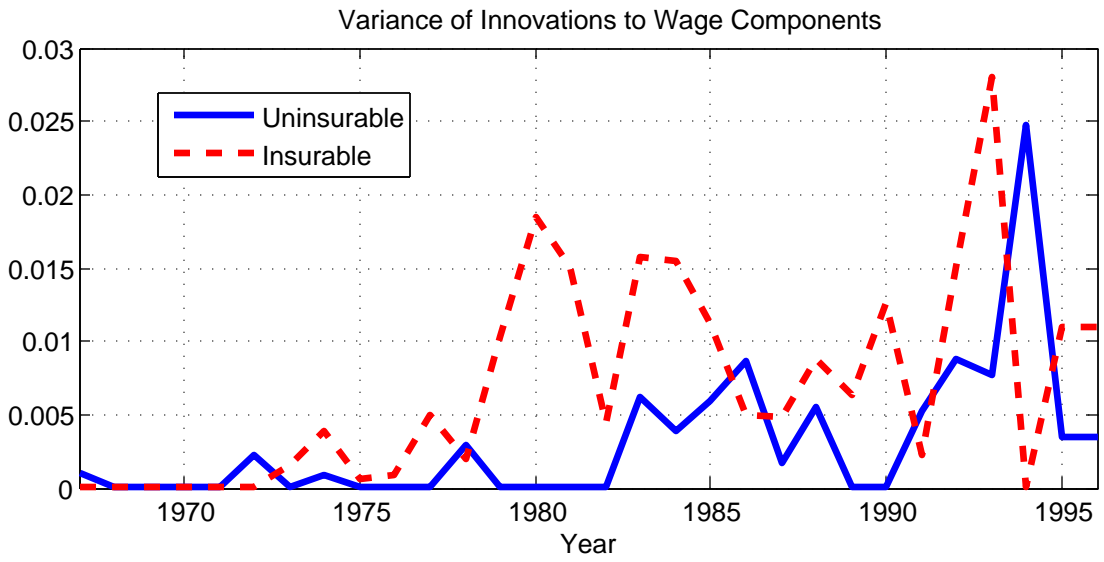
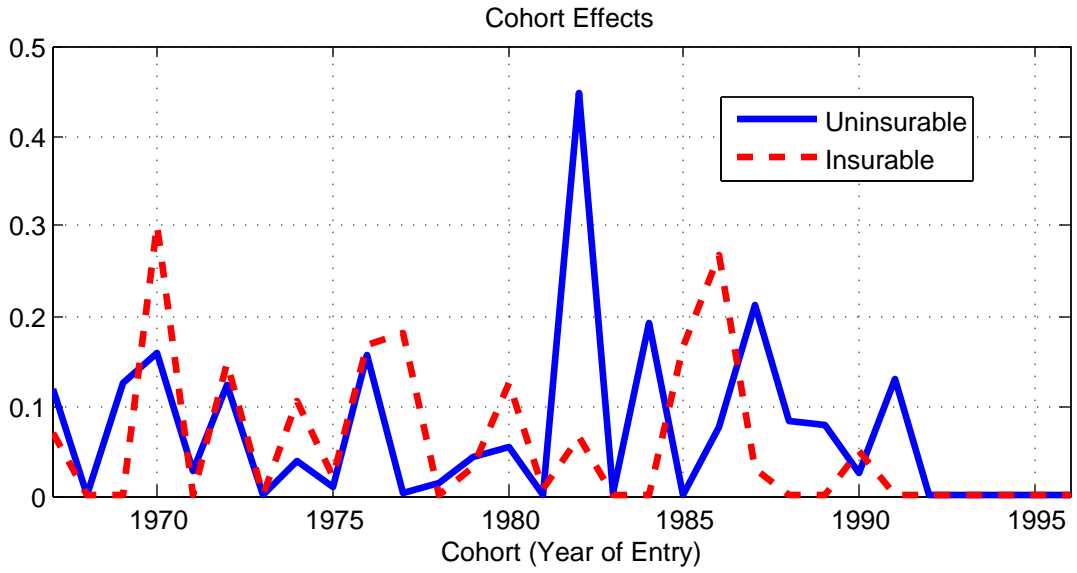
Covariance btw Changes in Wages and Hours by Year



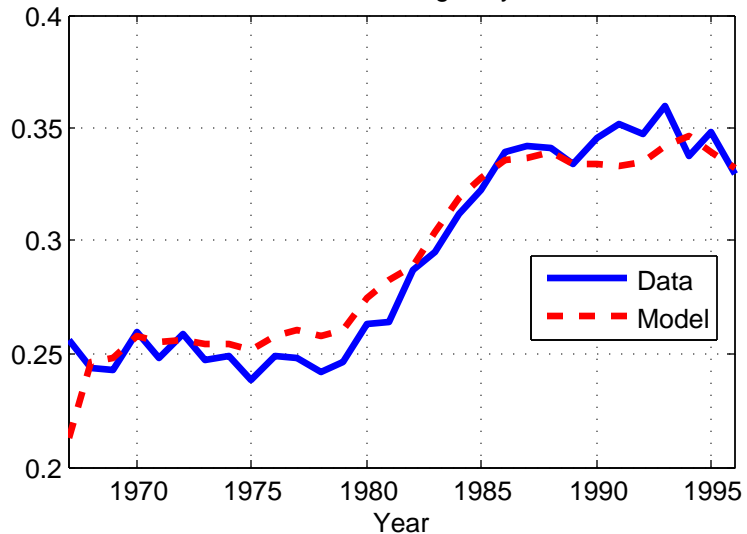
Parameter Estimates: Alternative Models

	γ	σ	$v_{\alpha 0}$	$v_{\varepsilon 0}$	v_{η}	v_{ω}
Benchmark	2.30	6.94	0.079	0.069	0.0058	0.0031
Cohort effects	2.18	6.12	0.071	0.058	0.0070	0.0031
No pref. het.	1.59	2.20	0.073	0.078	0.0048	0.0036

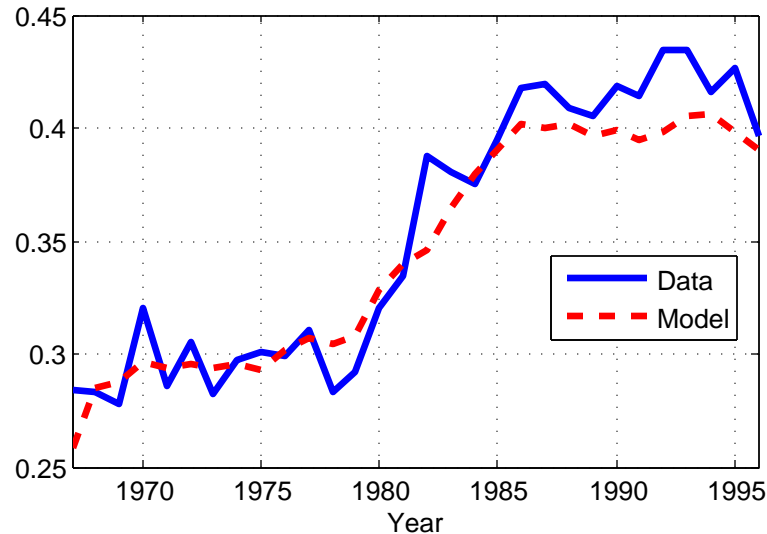
	v_{φ}	$v_{\alpha\varphi}$	v_{ζ}	$v_{\mu c}$	$v_{\mu y}$	$v_{\mu h}$
Benchmark	3.22	0.014	1.20	0.059	0.045	0.016
Cohort effects	2.62	0.016	0.91	0.058	0.044	0.016
No pref. het.	0.00	0.00	0.00	0.096	0.023	0.039



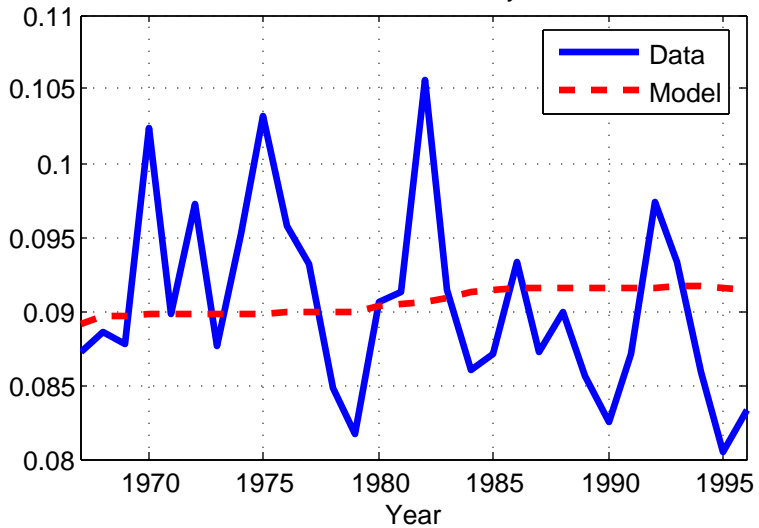
Variance of Wages by Year



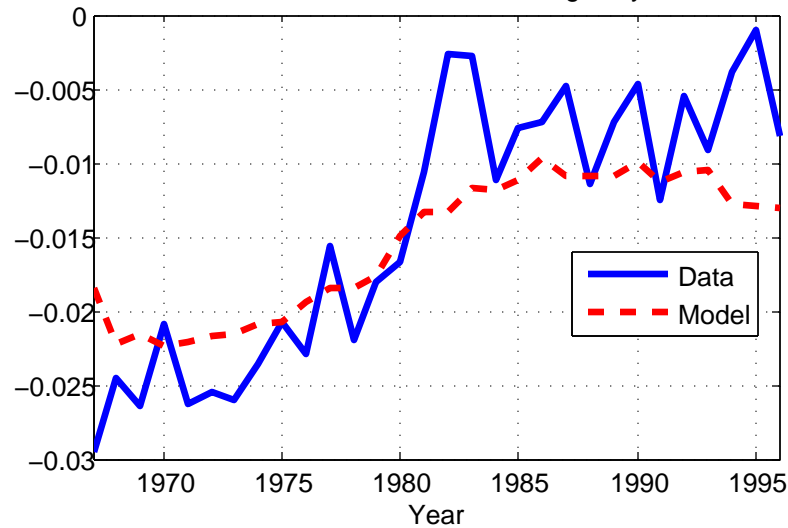
Variance of Earnings by Year



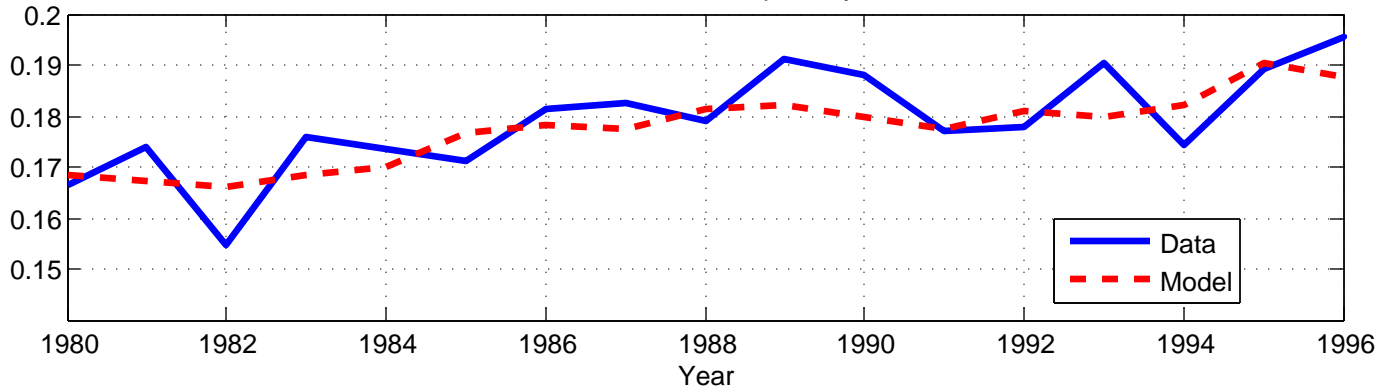
Variance of Hours by Year



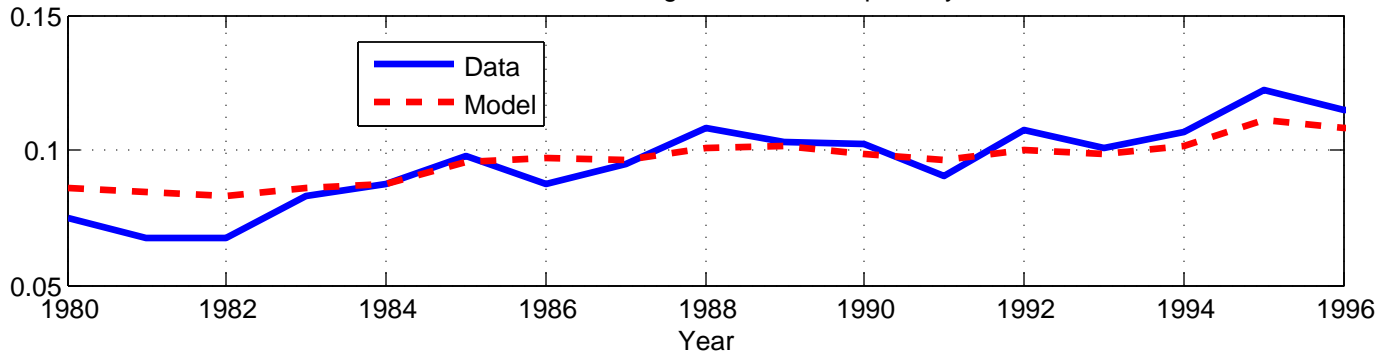
Covariance btw Hours and Wages by Year



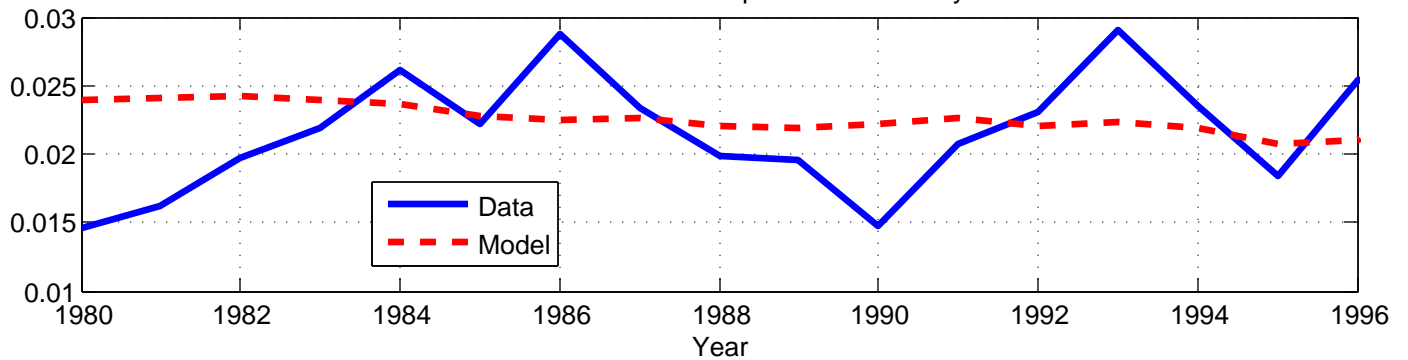
Variance of Consumption by Year

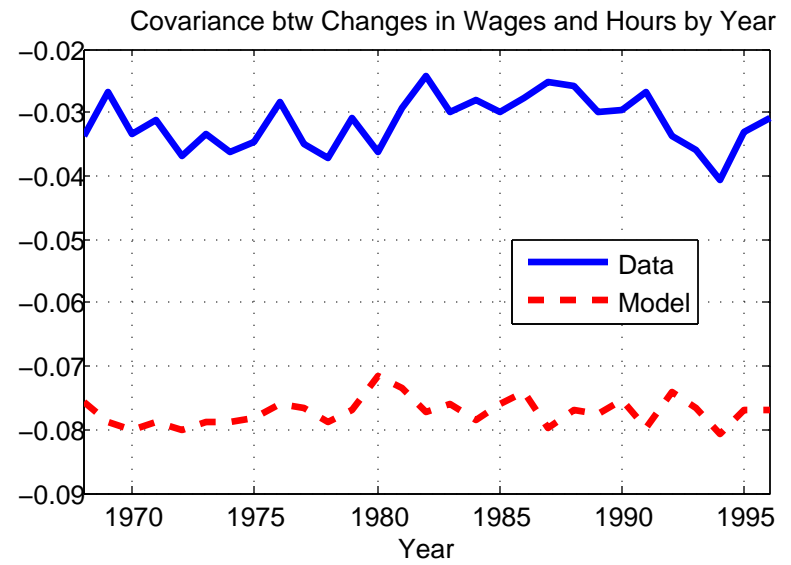
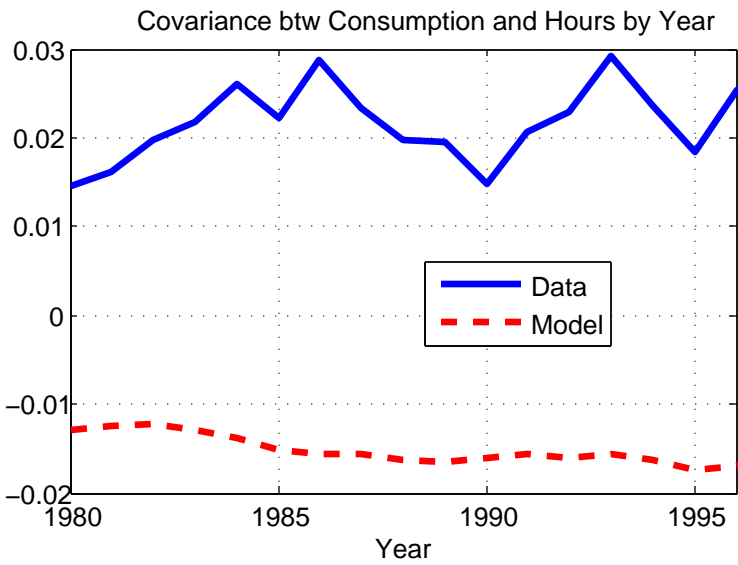
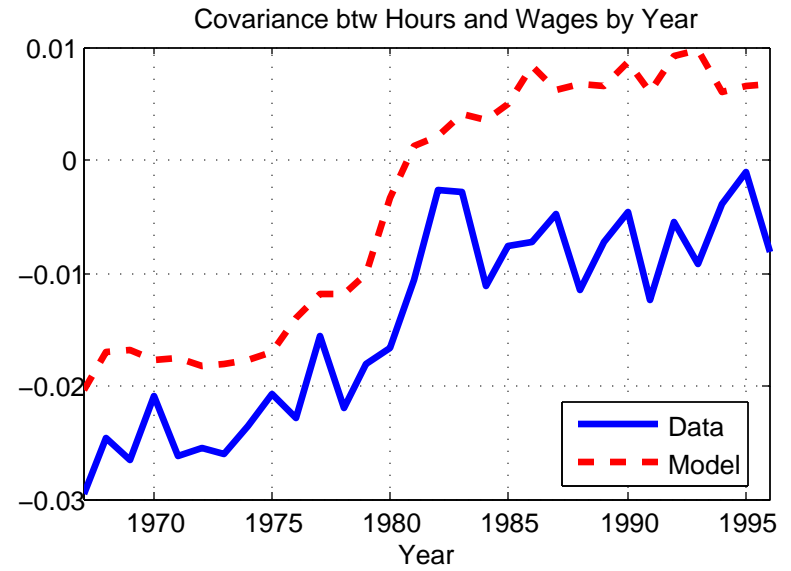
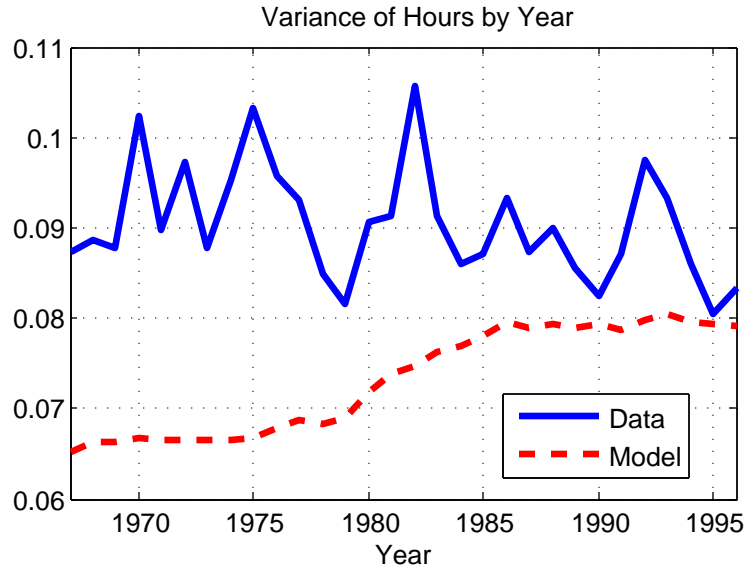


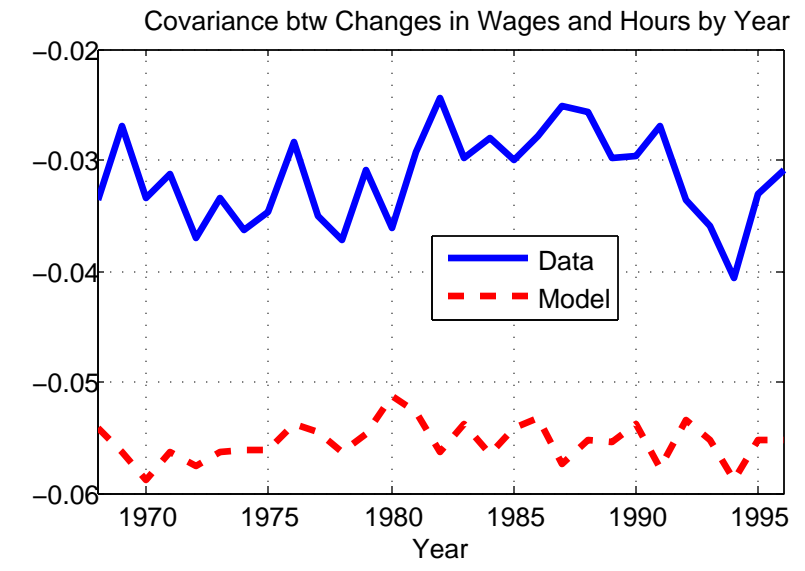
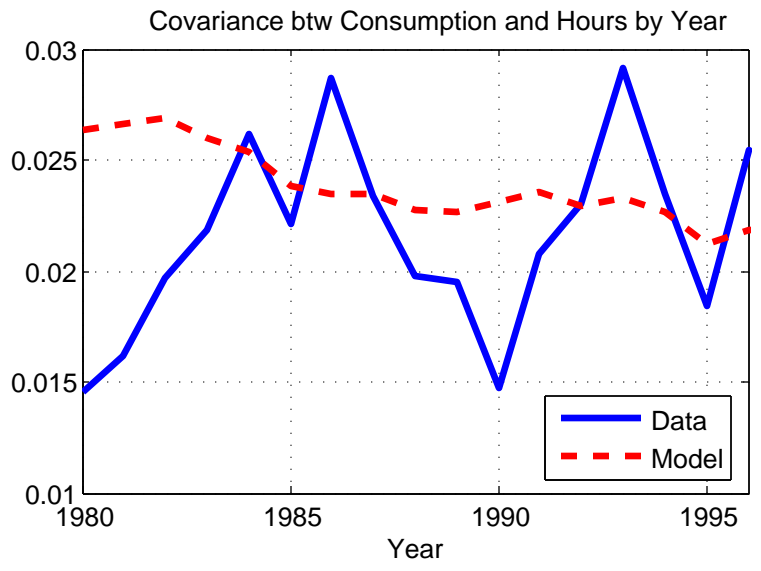
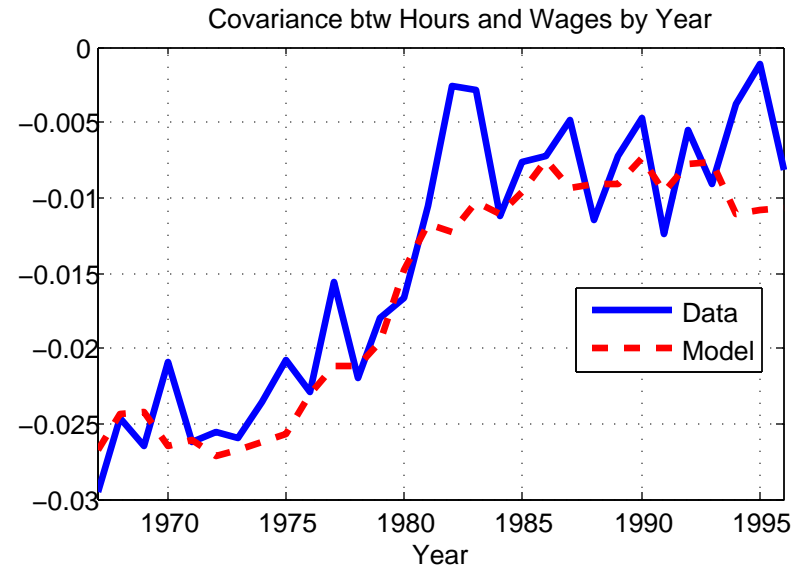
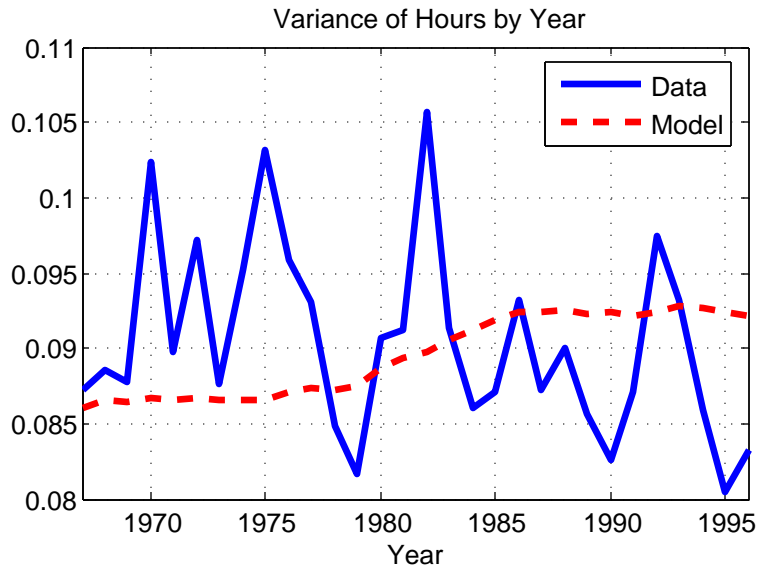
Covariance btw Wages and Consumption by Year



Covariance btw Consumption and Hours by Year







Conclusions

- Built life-cycle model that is rich enough to incorporate a realistic process for wage risk and some key sources of insurance
- Framework is tractable → simple to understand predictions for inequality
- Estimate model to offer positive account of the evolution of inequality by age and through time
 - Model can broadly account for the key patterns
 - Most of increase in wage dispersion over past thirty years insurable
 - Preference heterogeneity required to account for facts about hours