**Comparing models and data**

Suppose we would like to see whether output is more or less volatile than investment

We want a measure of volatility that is invariant to scale

Simple variance or standard deviation is not scale invariant

Suppose for example that investment is a constant fraction $\lambda$ of GDP

$$x = \lambda y$$

Now

$$sd(x) = \lambda sd(y)$$

A simple remedy would be rescale both variables by their mean prior to computing the standard deviation

$$sd(\frac{x}{\mu_x}) = \frac{1}{\mu_x} sd(x) = \frac{1}{\lambda \mu_y} \lambda sd(y) = sd(\frac{y}{\mu_y})$$

Now if we have variables that are non-stationary we need to apply some sort of filter prior to computing measures of dispersion.

Suppose $x_{gt}$ is the value of the growth component at $t$, and $x_{ct} = x_t - x_{gt}$ is the value of cyclical component

Now rather than dividing by the mean, we should divide by the trend prior to computing the standard deviation

The percentage standard deviation of the filtered series is given by

$$\% sd(x_{ct}) = 100 \times sd\left(\frac{x_{ct}}{x_{gt}}\right)$$

In our simple example, if our filter has the property that $x_{gt} = \lambda y_{gt}$, then we will get $\% sd(x_{ct}) = \% sd(y_{ct})$.

Note that

$$\frac{x_{ct}}{x_{gt}} = \frac{x_t - x_{gt}}{x_{gt}} \approx \log(x_t) - \log(x_{gt})$$

So a simpler way to compute percentage standard deviations is to

1. log the original series: $z_t = \log(x_t)$
2. filter the logged series: $z_t = z_{gt} + z_{ct}$
3. compute the standard deviation of the logged, filtered series: $100 \times sd(z_{ct})$.

This is the standard approach. However, if you occasionally have negative observations for a variable (so you cannot take logs), it is useful to remember the alternative formula. The two approaches will not give identical answers because (i) the log approximation is only correct for small deviations, and (ii) $\log(x_{gt}) \neq z_{gt}$ (the log of the trend is not the trend of the log).

The percentage standard deviation is sometimes called the relative standard deviation or the co-efficient of variation. When so named, it is usually not multiplied by 100.

Note that we have to filter non-stationary data.

In many cases, the output of our simulated model will be stationary.

Nonetheless, it is wise to apply the same filter to both model and data, since the filter is typically doing more than just guaranteeing stationarity, and we want to be sure to compare our real and artificial data consistently.