Homework 2, due in class on October 19th

International Finance 2005 - Heathcote

September 30th 2005

This homework is in two parts. The first part is mostly do-able without the computer. The second part involves linearizing the equations characterizing equilibrium around the non-stochastic steady state, and finding decision rules and laws of motion for state variables using the Schur decomposition method. It will be useful to make sure that we all agree on where the steady state lies before proceeding with this second part. Thus we will discuss this in class on October 12th, which will also be an opportunity to discuss any other problems you might run into.

1. Model

Consider the following economy. Two economies produce the same good, which may be used for consumption or investment. Capital and labor are immobile internationally. Each economy is populated by a continuum (of measure 1) of identical agents. The only asset traded internationally is a single non-contingent bond. Let s^t denote the state (history) of the world economy at date t.

Goods are produced by competitive firms, with production technologies given by

$$\begin{aligned} Y(s^t) &= e^{z(s^t)} K(s^{t-1})^{\theta} N(s^t)^{(1-\theta)} \\ Y^*(s^t) &= e^{z^*(s^t)} K^*(s^{t-1})^{\theta} N^*(s^t)^{(1-\theta)} \end{aligned}$$

where z denotes (log) total factor productivity, θ is capital's share in production, and stars denote foreign variables. Note that the notation convention is that variables are indexed by the date and state in which they were chosen; thus the capital available for production at date t is indexed by s^{t-1}. Productivity shocks follow a joint autoregressive process defined by

$$\begin{pmatrix} z(s^t) \\ z^*(s^t) \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} z(s^{t-1}) \\ z^*(s^{t-1}) \end{pmatrix} + \begin{pmatrix} \varepsilon(s^t) \\ \varepsilon^*(s^t) \end{pmatrix}$$

where

$$\begin{pmatrix} \varepsilon(s^t) \\ \varepsilon^*(s^t) \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\varepsilon^2 & \rho\sigma_\varepsilon^2 \\ \rho\sigma_\varepsilon^2 & \sigma_\varepsilon^2 \end{pmatrix} \right).$$

Firms rent capital and labor from households at rates $r(s^t)$ and $w(s^t)$. Capital depreciates at rate δ. Preferences for the domestic representative agent are given by

$$\sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi(s^t) \left[U\left(c(s^t), n(s^t)\right) - \chi B(s^t)^2 \right]$$

where

$$U\left(c(s^t), n(s^t)\right) = \log(c(s^t)) + \psi \log(1 - n(s^t))$$

β is the household's subjective discount factor and $\pi(s^t)$ is the unconditional time zero probability of history s^t. $B(s^t)$ denotes bond holdings at s^t; thus there is a quadratic penalty to borrowing or lending internationally. Preferences for the foreign agent are exactly the same, except that all arguments are starred.

Each period households decide how much labor to supply (taking as given $w(s^t)$) and how to split current period income from supplying capital and labor between current consumption and investment. The relation between investment $x(s^t)$ and the amount of capital available for production in the next period is given by

$$k(s^t) = (1-\delta)k(s^{t-1}) + \phi\left(\frac{x(s^t)}{k(s^{t-1})}\right)k(s^{t-1})$$

where

$$\phi\left(\frac{x(s^t)}{k(s^{t-1})}\right) = \left(\delta^* - \frac{\delta^*}{\gamma}\right) + \left(\frac{1}{\gamma\delta^{*(\gamma-1)}}\right)\left(\frac{x(s^t)}{k(s^{t-1})}\right)^{\gamma}$$

where δ^* is the steady state value for $\left(\frac{x(s^t)}{k(s^{t-1})}\right)$. Note that if $\gamma = 1$ then we have the standard capital accumulation equation. If $\gamma < 1$, the we have the standard accumulation equation only if $\left(\frac{x(s^t)}{k(s^{t-1})}\right) = \delta^*$.

One unit of the bond purchased at s^t has price $Q(s^t)$ and delivers one unit of consumption in the following period.

2. EXERCISES

1. Carefully define the household's problem and the firm's problem (for either the domestic or the foreign agent).

2. Carefully define an equilibrium for this economy.

3. List a set of difference equations that jointly characterize equilibrium for this economy.

4. Assume the following parameter values: $A_{11} = A_{22} = 0.9$, $A_{12} = A_{21} = 0.0$, $\sigma_\varepsilon^2 = 0.0001$, $\rho = 0$, $\theta = 0.36$, $\delta = 0.02$, $\gamma = 0.95$, $\beta = 0.99$, $\psi = 2$, $\chi = 0.000001$.

5. Solve for the steady state of this economy, given the above parameter values.

6. Linearize the equations characterizing equilibrium around the non-stochastic steady state.

7. Apply the Schur decomposition to this system of equations. Set the parameter that indicates the dividing line between stable and unstable roots to a number slightly greater than 1 (e.g. 1.00001)

8. Verify that if the utility cost on bond holdings is positive, then the number of stable roots is equal to the number of pre-determined state variables

9. Verify that as $\chi \to 0$ one of the previously stable roots becomes a unit root.

10. Simulate the model economy 200 times, each simulation being 100 periods in length for the following configurations of parameter values. For each case compute (i) cross-country correlations for productivity, output, consumption, investment and hours, (ii) the correlation between domestic output and net exports, (iii) percentage standard deviations for output and investment, and (iv) the average absolute value for the ratio of net exports to output..

    (a) Case 1: Baseline parameter values described above.
    (b) Case 2: Smaller capital adjustment costs: Case 1 but with $\gamma = 0.99$
    (c) Case 3: Larger bond-holding costs: Case 1 but with $\chi = 0.01$
    (d) Case 4: More persistent shocks: Case 1 but with $A_{11} = A_{12} = 0.99$
    (e) Case 5: Negative spill-overs: Case 1 but with $A_{12} = A_{21} = -0.08$

11. In this economy, asset markets are not complete. It is nonetheless of interest to ask how much risk sharing can be achieved by trading a single bond. Recall that if asset markets were complete, perfect risk pooling would imply equalizing the marginal utility of consumption across countries in every date and state.

    (a) Simulate the model, given the Case 1 parameter values, for 100 periods, drawing producitivity shocks according to the previously specified process.
    (b) Plot, on the same graph, the marginal utility of consumption across the simulation for both the domestic and the foreign agent.
    (c) Repeat the same exercize for the Case 3 parameter values. What do the differences between the two graphs tell us?

12. Can you find any combination of parameter values that gives $corr(y_1, y_2) > corr(c_1, c_2)$, $corr(nx, y_1) < 0$, and $corr(x_1, x_2)$, $corr(n_1, n_2) > 0$?