Brief Notes on "Why doesn't capital flow from rich to poor countries?" Lucas AER 1990 80/2 and "Do migration restrictions matter?" Klein and Ventura, mimeo 2005

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September 2005

Capital flows into rich countries are larger than into poor countries.

Some countries have much higher output per worker than others.

In 2002, according to OECD, at PPP exchange rate, GDP per capita in US: \$35,750, in India \$2,670 (ratio > 13)

Suppose production technology is

$$Y = AK^{\theta}(hN)^{1-\theta}$$

where K is capital, N is the number of workers, h is human capital per worker (hN is effective labor input) and A is TFP.

So output per worker is

$$y = Ak^{\theta}h^{1-\theta}$$

Output per worker can differ across countries because of differences in A differences in k or differences in h

1. Lucas says: suppose it was all differences in k. Consider a two country world, where $h_1 = h_2$ and $A_1 = A_2$

$$\frac{y_1}{y_2} = \frac{A_1 k_1^{\theta} h_1^{1-\theta}}{A_2 k_2^{\theta} h_2^{1-\theta}} = \frac{k_1^{\theta}}{k_2^{\theta}}$$

$$\frac{k_1}{k_2} = \left(\frac{y_1}{y_2}\right)^{\frac{1}{\theta}}$$

This would then imply

$$\frac{r_1}{r_2} = \frac{\theta A_1 k_1^{\theta-1} h_1^{1-\theta}}{\theta A_2 k_2^{\theta-1} h_2^{1-\theta}} = \frac{k_1^{\theta-1}}{k_2^{\theta-1}} = \left(\frac{y_1}{y_2}\right)^{\frac{\theta-1}{\theta}}$$

Let country 1 be the US, and country 2 be India. Suppose $\theta = 1/3$ and $y_1/y_2 = 15$.Then $r_1/r_2 = 15^{-2} = 1/225$ – the return to capital in India should be 225 that in the US! Hence the question in the paper. Potential answers: risk of non-repayment, monopoly lender, capital controls.

2. What if it was all differences in h? Suppose $A_1 = A_2$ and capital is mobile internationally, so that $r_1 = r_2$. The assumption of capital mobility implies

$$\begin{aligned}
\theta A_1 k_1^{\theta-1} h_1^{1-\theta} &= \theta A_2 k_2^{\theta-1} h_2^{1-\theta} \\
k_1^{\theta-1} h_1^{1-\theta} &= k_2^{\theta-1} h_2^{1-\theta} \\
\frac{k_1}{h_1} &= \frac{k_2}{h_2}
\end{aligned}$$

The wage per effective unit of labor in country i is

$$\begin{aligned}
w_i &= A_i(1-\theta)K_i^{\theta}(h_i N_i)^{-\theta} \\
&= A_i(1-\theta)k_i^{\theta}(h_i)^{-\theta}
\end{aligned}$$

The wage for a worker with a given level of human capital x working in country i is

$$xA_i(1-\theta)\left(\frac{k_i}{h_i}\right)^{\theta}$$

Note that since the ratio $\frac{k_i}{h_i}$ is equalized across countries, this worker will earn the same in both countries. But this runs contrary to evidence (1) that migrants see their earnings rise - Lucas cites some evidence that they rise by a factor of around 5 for immigrants from India to the US and (2) lots of people want to move.

3. What if it was all differences in A? (Gervais and Klein will assume this looking at EU enlargement). In this case, assuming perfect capital mobility, equalization of the return to capital implies

$$\begin{aligned}
\theta A_1 k_1^{\theta-1} h_1^{1-\theta} &= \theta A_2 k_2^{\theta-1} h_2^{1-\theta} \\
A_1 k_1^{\theta-1} &= A_2 k_2^{\theta-1} \\
\frac{A_1}{A_2} &= \left(\frac{k_1}{k_2}\right)^{1-\theta} \\
\frac{y_1}{y_2} &= \frac{A_1 k_1^{\theta} h_1^{1-\theta}}{A_2 k_2^{\theta} h_2^{1-\theta}} = \frac{A_1}{A_2}\left(\frac{A_1}{A_2}\right)^{\frac{\theta}{1-\theta}} = \frac{A_1}{A_2}^{\frac{1}{1-\theta}}
\end{aligned}$$

So in the US India case we get

$$\frac{A_1}{A_2} = 15^{2/3} = 6.1$$

In this case the ratio of wages across countries is

$$\frac{w_1}{w_2} = \frac{A_1(1-\theta)k_1^{\theta}(h_1)^{-\theta}}{A_2(1-\theta)k_2^{\theta}(h_2)^{-\theta}} = \frac{A_1 k_1^{\theta}}{A_2 k_2^{\theta}} = \left(\frac{A_1}{A_2}\right)^{\frac{1}{1-\theta}} = \frac{y_1}{y_2}$$

Someone moving from India to the US can increase his wage by a factor of 15.

4. What if it is a mix of differences in A and h? The wage for a worker with a given level of human capital x working in country i is

$$xA_i(1-\theta)\left(\frac{k_i}{h_i}\right)^\theta$$

The ratio of what this worker can earn in country 1 relative to country 2 is

$$\kappa = \frac{xA_1(1-\theta)\left(\frac{k_1}{h_1}\right)^\theta}{xA_2(1-\theta)\left(\frac{k_2}{h_2}\right)^\theta} = \frac{A_1\left(\frac{k_1}{h_1}\right)^\theta}{A_2\left(\frac{k_2}{h_2}\right)^\theta}$$

Now

$$\frac{y_1}{y_2} = \frac{A_1 k_1^\theta h_1^{1-\theta}}{A_2 k_2^\theta h_2^{1-\theta}} = \frac{A_1 k_1^\theta h_1^{-\theta}}{A_2 k_2^\theta h_2^{-\theta}}\frac{h_1}{h_2} = \frac{w_1}{w_2}\frac{h_1}{h_2} = \kappa\frac{h_1}{h_2}$$

So if $\frac{y_1}{y_2} = 15$ and $\kappa = 5$ then $\frac{h_1}{h_2} = 3$. Interest rate equalization implies

$$\begin{aligned}
\theta A_1 k_1^{\theta-1} h_1^{1-\theta} &= \theta A_2 k_2^{\theta-1} h_2^{1-\theta} \\
\frac{A_1}{A_2} &= \frac{\left(\frac{k_1}{h_1}\right)^{1-\theta}}{\left(\frac{k_2}{h_2}\right)^{1-\theta}}
\end{aligned}$$

So

$$\begin{aligned}
\kappa &= \frac{A_1}{A_2}\left(\frac{A_1}{A_2}\right)^{\frac{\theta}{1-\theta}} = \frac{A_1}{A_2}^{\frac{1}{1-\theta}} \\
\frac{A_1}{A_2} &= \kappa^{1-\theta} = 2.9
\end{aligned}$$

So

$$\frac{k_1}{k_2} = \left(\frac{A_1}{A_2}\right)^{\frac{1}{1-\theta}}\left(\frac{h_1}{h_2}\right) = \kappa\left(\frac{y_1}{y_2}\right)/\kappa = 15$$

5. Klein and Ventura production technology

$$Y = AK^\lambda (hL)^\eta F^{1-\lambda-\eta}$$

so

$$y = Ak^\lambda h^\eta f^{1-\lambda-\eta}$$

Interest rate equalization implies

$$r = \lambda A_1 K_1^{\lambda-1}(h_1 L_1)^\eta F_1^{1-\lambda-\eta} = \lambda A_2 K_2^{\lambda-1}(h_2 L_2)^\eta F_2^{1-\lambda-\eta}$$

$$A_1 k_1^{\lambda-1} {h_1}^{\eta} f_1^{1-\lambda-\eta} = A_2 k_2^{\lambda-1} {h_2}^{\eta} f_2^{1-\lambda-\eta}$$

$$\left(\frac{k_1}{k_2}\right)^{\lambda} = \left(\frac{A_1 {h_1}^{\eta} f_1^{1-\lambda-\eta}}{A_2 {h_2}^{\eta} f_2^{1-\lambda-\eta}}\right)^{\frac{\lambda}{1-\lambda}}$$

$$\frac{y_1}{y_2} = \left(\frac{A_1 h^{\eta} f^{1-\lambda-\eta}}{A_2 h^{\eta} f^{1-\lambda-\eta}}\right)\left(\frac{A_1 {h_1}^{\eta} f_1^{1-\lambda-\eta}}{A_2 {h_2}^{\eta} f_2^{1-\lambda-\eta}}\right)^{\frac{\lambda}{1-\lambda}} = \left(\frac{A_1}{A_2}\right)^{\frac{1}{1-\lambda}} \left(\frac{h_1}{h_2}\right)^{\frac{\eta}{1-\lambda}} \left(\frac{f_1}{f_2}\right)^{\frac{1-\lambda-\eta}{1-\lambda}}$$