The Great Resignation and Optimal Unemployment Insurance

Zhifeng Cai and Jonathan Heathcote*

March 20, 2023

Abstract

How generous should social insurance be when quits are an important driver of transitions into non-employment? We address this question using a multi-sector directed search model extended to incorporate endogenous quits both to other jobs and to non-employment. We find that unemployment insurance is optimally much less generous in an economy with quits than in one without. Workers quit too often in the competitive equilibrium, and private markets co-ordinate on excessively high "efficiency" wages. We find that a decline in vacancy posting costs is consistent with the observed increase in quits in the United States since the Great Recession, and that such a change calls for a reduction in UI generosity.

1 Introduction

In most labor search models, the rate at which workers transition to unemployment is exogenous. Optimal unemployment insurance design has focused on the trade-off between consumption smoothing versus preserving job-search incentives. However, most jobs end

*Cai: Department of Economics, Rutgers University, 75 Hamilton St, New Brunswick, NJ 08901, USA., (email: zhifeng.cai@rutgers.edu). Heathcote: Research Department, Federal Reserve Bank of Minneapolis, 90 Hennepin Ave., Minneapolis, MN 55401, USA., (email: heathcote@minneapolisfed.org). We thank seminar participants at the Minneapolis Fed, Rutgers, the Philadelphia Fed, University of Hong Kong, Baptist University of Hong Kong, City University of Hong Kong, and the SED for helpful comments. We also thank David Wiczer, Serdar Birinci, Derek Neal, Christopher Moser, and Giuseppe Moscarini for very useful conversations. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis, or the Federal Reserve System.

because the worker quits, not because the worker is laid off. Some of these quits are immediate job to job transitions. But a large and growing share of quits are not immediately followed by a new job.

We extend a directed search and matching model to incorporate quits. Workers in our model are subject to idiosyncratic shocks to the disutility of work, and may choose to quit to non-employment when these shocks are large. In addition we allow for on-the-job search, so a portion of quits involve immediate transitions to a new job.

To start, we assume that the government cannot differentiate between workers who are non-employed because they were laid off versus those who chose to quit. Workers are risk-averse, while firms are risk-neutral. Firms have commitment and can commit to dynamic wage contracts. Crucially, firms do not observe worker preference shocks and cannot directly verify the existence of outside offers. Our focus is on understanding how quits affect the optimal level of unemployment insurance.

We show that in this setting, workers tend to quit inefficiently often, breaking up matches which have positive joint surplus. In equilibrium, this translates into workers directing search toward high wage jobs: high "efficiency" wages partially mitigate the excessive quitting problem. In addition, firms offer wage contracts with two features designed to reduce quitting. First, wages rise with tenure: backloading wage payments reduces incentives to quit to non-employment. Second, firms stochastically match outside offers, which reduces the rate at which workers quit to take other jobs.

We show that optimal policy addresses the excessive quitting inefficiency by reducing the optimal UI replacement rate, relative to the rate that would be optimal without the quit margin. We illustrate this result most starkly in a simple static version of the model in which workers are risk neutral and there is no role for any policy intervention absent the quitting margin (Acemoglu and Shimer, 1999). Once that margin is introduced, the optimal transfer to non-workers becomes negative.

A more favorable interpretation of quitting behavior is that it reflects workers trying to improve match quality. Perhaps policy should not seek to discourage quitting if workers need to sample multiple jobs in order to find a good match. The quantitative version of our model features stochastic match quality, and we replicate the observed rate of job-to-job transitions, and the average wage growth associated with those transitions. While on the job search is one way to escape a bad match, quitting to unemployment is a back-up strategy for workers who fail to match, or who draw another low quality match. Indeed, most quitters to non-employment in our model are in low quality matches. Because reducing UI lowers the quit rate, it also therefore slows down the rate at which workers reallocate from bad to good matches. The design of optimal UI policy in our quantitative model therefore takes into account both the inefficiency and reallocation aspects of quitting behavior.

We set the model vacancy posting cost, the match efficiency parameter, and the average utility cost of work to match observed unemployment, the job opening rate, and the quit rate. To calibrate the cross-sectional variance of the utility cost of work, we exploit cross-industry variation in wages and quit rates. To the extent that quits are heavily concentrated in low wage industries (such as food services) the model indicates a large average utility cost of work, but modest cross-worker dispersion in that cost.

Our main quantitative findings are as follows. In our baseline calibration, the optimal UI replacement rate is 38.4 percent. In a version of the model in which the variance of preference shocks is very small, so that the equilibrium quit rate is near zero, the corresponding optimal replacement rate is 48.9 percent. Thus, incorporating quitting has a big impact on the optimal policy.

Suppose the government can differentiate between non-workers who quit versus those who were fired. How much more or less generous should transfers be for the quitters versus those were laid off? In our baseline calibration, we find a 48.5 percent optimal replacement rate for the former group, versus a much less generous rate of 29.5 percent for the latter group. That suggests the government should penalize quitters, as a way to discourage excessive quitting. However, we find only a small welfare gain of 0.3 percent of consumption associated with moving from a universal non-worker benefit of 38.4 percent to this differentiated quit-versus-fired benefit model. To the extent that it is costly to elicit information about precisely how particular workers came to be non-employed, a universal benefit might be preferable.

We also use the model to interpret the rise in the quit rate between 2006 (the end of

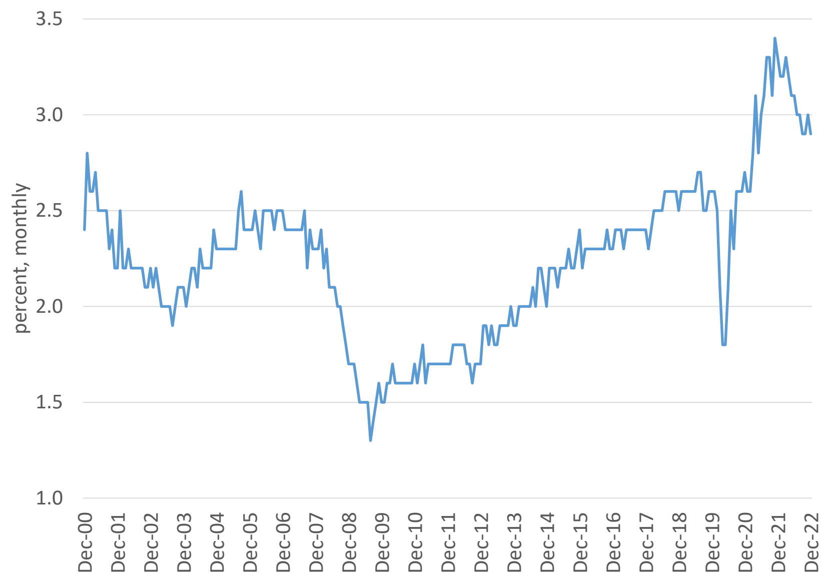

Figure 1: JOLTS Quit Rate

the pre-Global Financial Crisis expansion) and the summer of 2022. Data from JOLTS indicate much higher quit and job opening rates in 2021 relative to 2006, but a similar unemployment rate. The model can broadly replicate observed changes in unemployment, job openings and quits given a decline in the cost of posting vacancies. One possible interpretation is that new online platforms like Monster and Indeed have made it cheaper for firms to contact workers. We find that a decline in the vacancy posting cost reduces the optimal UI replacement in our model, because workers will quit more readily if they know that new jobs are easier to find. This translates into a two percentage point reduction in the optimal UI replacement rate between 2006 and 2021.

1.1 Literature

The paper is related to two strands of literature, one on optimal unemployment insurance design, and another on quits.

There is a large literature on optimal unemployment insurance. Baily (1978) and Chetty (2006) framed a trade-off between UI helping risk-averse workers smooth consumption

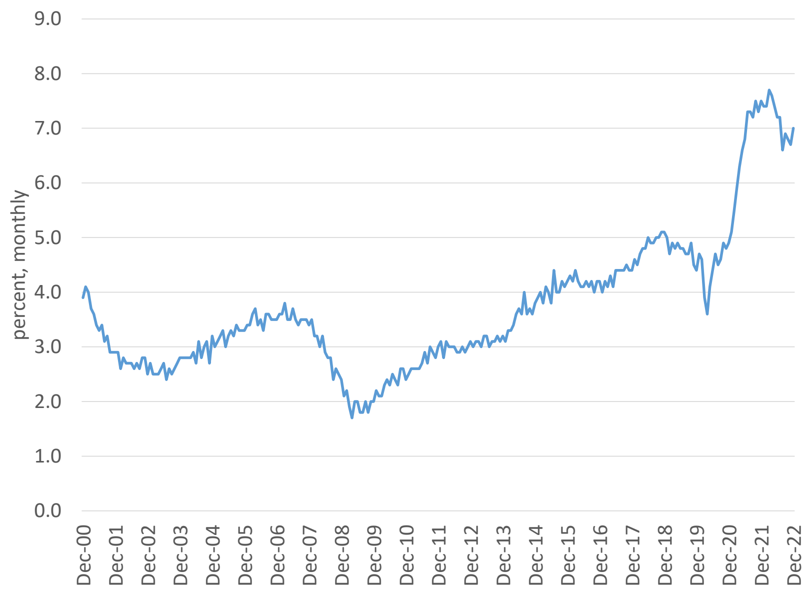

Figure 2: JOLTS Job Openings Rate

while unemployed, while reducing incentives for job search and thus raising unemployment. In our setting, optimal UI design also involves a trade-off between consumption risk and the equilibrium level of consumption. But we emphasize a range of different channels via which the generosity of benefits impacts average consumption. Most of the literature emphasizes the negative impact of UI on job search. In our directed search framework, more generous UI makes searching workers more picky, raising the average duration of non-employment spells. But we emphasize other channels too: more generous UI encourages quits, thereby reducing average output, and also affects average match quality and thus productivity.

Acemoglu and Shimer (1999) was one of the first papers to study optimal unemployment insurance in a directed search setting with risk-averse workers. Golosov et al. (2013) develop additional insights on the nature of optimal policy in a similar setting. Relative to those papers, our key innovation is to emphasize the impact of quitting – both to non-employment and to other jobs – on optimal policy design in a quantitative environment.

There are many papers modeling quits, but these mostly focus on workers leaving their current jobs for another one, so-called "job-to-job transitions". Early examples include Shimer (2006) in random search models and Delacroix and Shi (2006) in a directed search setting (see also Shi 2009 and Menzio and Shi 2011). More recently, Mercan and Schoefer (2020) and Elsby et al. (2022) explore the notion of "vacancy chains", illustrating the interactions between workers' quitting behavior and firm's replacement hiring, and how such interactions can lead to amplification of labor market fluctuations.

Regarding how firms can attempt to reduce quitting, Stevens (2004) and Burdett and Coles (2003) were the first to recognize that backloading compensation can reduce quitting. Shi (2009) shows that that insight also applies in a directed search setting. Balke and Lamadon (2022) find that firms also want to backload wages when there is a moral hazard friction such that low worker effort can lead to job destruction.

Besides backloading wages, we also give firms a second tool to reduce quitting, which is to match outside offers. A common assumption in the literature (e.g., in Shi 2009) is that firms do not respond to outside offers. One motivation for this assumption has been that outside offers are typically not verifiable.[1] However, Moore (1985) shows that stochastic contracts can incentivize truthful reporting of a privately observed worker reservation wage. We build on that insight in modeling stochastic offer matching, whereby firms either match a reported outside offer or fire the worker, and show that this is more profitable than simply ignoring such offers.[2]

Recently, an emerging literature studies models where workers can quit into non-employment, a margin that became more relevant in the Great Resignation era. Blanco et al. (2023) consider an environment where workers quit due to productivity variations and wage rigidities, and explore the impact of monetary policy shocks. Qiu (2022) studies the business cycle implications of the quitting channel. Neither of those papers studies

[1]Burdett and Coles (2003) write: "An important assumption ... is that a firm does not respond to outside offers received by any of its employees. Clearly this restriction is not satisfied in some labor markets such as the academic labor market in the U.S. Nevertheless, there are reasons to suspect our restriction holds in other labor markets, especially those markets where workers are homogeneous. First, outside offers may not be observable by firms. Indeed, why should a firm verify to another firm that it has made a particular offer to a worker? Of course given offers from other firms are not observed, they will be ignored."

[2]Outside offers can also be matched in the Postel–Vinay and Robin (2002) model. But note that theirs is a complete information setup, in which the details of outside offers are fully observable. In this tradition, Elsby and Gottfries (2021) consider a setting where firms can match offers with an exogenously-specified probability.

the optimal design of labor market policies, which is the focus of this paper.

In parallel with the quantitative theoretic literature on optimal UI design, a separate set of papers has attempted to quantify empirically the impact of UI benefits on various labor market flows and stocks. This is hard, because exogenous changes in benefit generosity are rare. Johnston and Mas (2018) and Karahan et al. (2022) exploit an unexpected cut in maximum UI duration in Missouri in 2011, and find significant declines in non-employment duration, and a large rise in market tightness. What evidence is there on the drivers of quits to non-employment? As we will show, quit rates decline quite steeply with earnings, suggesting that economic considerations are important in quitting decisions. In addition, fewer workers quit in recessions, suggesting that many quitters plan to return to work, and are hesitant to quit when doing so is hard. During Covid, UI benefits were made more generous, and eligibility criteria were temporarily relaxed, increasing the odds that quitters would receive benefits.[3] In fact, between May and September 2020 the number of workers receiving unemployment benefits exceeded the number reporting being unemployed. Ganong et al. (2022) find that the expansion of benefits had a small negative impact on the job-finding rate, but argue that that was because they were introduced at a time when the job finding rate was already depressed and thus could not fall much.

2 Model

Time is discrete and the horizon is infinite. There are no aggregate shocks. There is a unit mass of infinitely-lived workers. The economy is composed of different sectors indexed by n. Workers are ex ante heterogeneous with respect to the sector to which they belong, and cannot move across sectors. Workers start each period either matched to a firm or unmatched. Matched workers are further differentiated by the quality of their match z, where match quality is drawn at the time a new match is formed, and remains fixed for the duration of the match. We assume two possible values for match quality, $z \in \{z_H, z_L\}$,

[3] For example, the CARES Act provided Pandemic Unemployment Assistance to "primary caregivers" of children at home due to school closures."

which are drawn with probabilities μ_H and $\mu_L = 1 - \mu_H$. A worker in sector n with match quality z produces zY_n units of output each period for as long as the match survives.

At the start of each period, both matched and unmatched workers search for new matches. We model the labor market in the directed search tradition. Firms post vacancies in sub-markets indexed by (i) sector n, (ii) the details of the employment contract promised to new matches, and (iii) the characteristics of the workers searching: whether they are matched or unmatched, and, for matched workers, the characteristics of their existing matches. There are two ways a match can end. First, each period every match is destroyed with fixed exogenous probability $1 - \gamma$. Exogenous match destruction shocks could be interpreted as reflecting large negative shocks to match productivity. The second way a match can end is if the worker chooses to quit, either to move to another job, or to transit to non-employment.

Workers have concave time-separable utility over consumption, c_{it}, and a disutility cost of work, χ_{it}. Workers and firms discount at a common rate β. Expected lifetime utility at date 0 for an individual i is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_{it}) - \chi_{it} \right)$$

where $u(.)$ is a concave function. If an individual works in period t, they are paid a wage w_{it}, and wage income is taxed at a constant rate τ. We will describe wage determination shortly. If an individual does not work, they receive a benefit $b(n_i)$ that depends on i's sector. Workers have no access to private insurance or credit markets. Thus

$$c_{it} = \begin{cases} w_{it}(1-\tau) & \text{if } i \text{ is working at } t \\ b(n_i) & \text{if } i \text{ is not working at } t \end{cases}$$

The utility cost of work is idiosyncratic and stochastic, and drawn independently each period from a distribution F, where F is common across sectors. Note that the utility cost is only paid in periods in which the individual is working.

Firms are risk neutral and seek to maximize profits. They post vacancies at a sector-specific cost ϕ_n. Firms observed match quality once a match is formed, but do not observe preference shocks χ, nor do they observe the worker's on-the-job search behavior or their receipt of outside offers. Firms can commit to flexible dynamic contracts that specify how wages will vary with the realization of match quality and with tenure. In contrast, workers cannot commit to future job search or quitting choices. We will formalize the firm problem recursively, and summarize the state of a particular firm-worker match by the pair (V, z) where V is the expected present of utility promised to the worker and z is the match quality. Before describing more details of how promised values and wages evolve, we first summarize the sequence of events during a period.

1. Workers begin each period either matched or unmatched. If a worker is matched, their state is (V, z).

2. All workers choose a submarket toward which they direct job search. The number of matches in a submarket is given by a function $m(u, v)$ where u is the number of workers searching, and v is the number of vacancies posted in the submarket. We assume that the matching function is constant returns to scale, so the probability that an searching worker is matched is given by $p(\theta) = m(1, \theta)$ where $\theta = v/u$ denotes market tightness. Conversely the probability that a vacancy translates into a match is given by $q(\theta) = p(\theta)/\theta$.[4]

   (a) Unmatched workers search in submarkets indexed by sector n, by promised value V^s, and by market tightness θ.

   (b) Matched workers search in submarkets indexed by the characteristics of the incumbent match (V, z), in addition to the triple (n, V^s, θ).

3. Workers who started the period matched report to their current employer whether or not they have received an outside offer. If the worker reports an offer, the incumbent firm either matches the expected value of the offer V^s and the worker stays with the incumbent firm or else the incumbent firm tells the worker to leave the firm. Offer-matching probabilities are specified as part of the dynamic wage-employment contract that firms offer to new hires, as described below, and incentivize truthful

[4] More precisely, the probability that an unmatched worker matches is $\max\{p(\theta), 1)\}$.

reporting in equilibrium. Thus, when the worker finds an outside offer and the incumbent firm does not match there is an E to E transition.

4. New matches draw a match quality z, which is immediately observed by both the firm and the worker.

5. With probability $(1-\gamma)$ each match is exogenously destroyed, generating an involuntary separation and what we label an E to U transition.

6. Workers who remain matched draw an idiosyncratic cost-of-work shock χ. Given their realization of this shock a worker chooses whether or not to quit. If a worker does quit, we label the transition E to N.[5]

7. Workers who remain matched produce, and all workers consume.

We now describe in more detail the search stage and the firm's dynamic contracting problem. To simplify notation, we temporarily suppress the sector notation.

Consider the vacancy posting problems for firms which post in markets for initially unmatched workers and for matched workers. For any possible submarket for unmatched workers indexed by (θ, V^s), let $J^u(\theta, V^s)$ denote the expected profit from posting an additional vacancy. Let $E\left[\Pi\left(V^s\right)\right]$ denote the maximum expected present value of profits for a firm when exiting the search and matching stage newly matched to a worker to whom the firm has promised V^s, prior to match quality being revealed. The expected profit from posting one more vacancy is the probability that vacancy translates into a match times expected profits conditional on a match minus the vacancy posting cost, i.e.,

$$J^u(\theta, V^s) = q\left(\theta\right) E\left[\Pi\left(V^s\right)\right] - \phi.$$

Free entry on the firm side implies $J(\theta, V^s) \leq 0$ for all submarkets (θ, V^s).

For any possible submarket for matched workers indexed by (θ, V^s, V, z), let $J^m(\theta, V^s, V, z)$ denote the profit from posting an extra vacancy. For firms posting vacancies in markets for matched workers, only meetings in which the incumbent firm does not match the outside

[5] Those non-participant N workers will transit into the U state at the beginning of the next period, given that they will start looking for jobs in the same way as other unmatched workers.

offer translate into new hires. Let $\bar{\zeta}\left(V^s, V, z\right)$ denote the probability that an incumbent firm that has promised V with match quality z matches an outside offer of V^s. The payoff from posting an additional vacancy is now

$$J^m(\theta, V^s, V, z) = q\left(\theta\right)\left(1 - \bar{\zeta}\left(V^s, V, z\right)\right)E\left[\Pi\left(V^s\right)\right] - \phi.$$

Free entry implies $J^m(\theta, V^s, V, z) \leq 0$ for all submarkets.

We conceptualize unmatched workers as being able to direct search to any sub-market (θ, V^s), subject to two constraints: (i) $J^u(\theta, V^s) \geq 0$, implying that firms are willing to post vacancies into the market, and (ii) $J^u(\theta, V^s) \leq 0$ implying that free entry on the firm side will erode strictly positive profits. Thus, unmatched workers solve

$$\max_{V^s, \theta}\left\{p\left(\theta\right)V^s + \left(1 - p\left(\theta\right)\right)V^u\right\}$$

subject to $J^u(\theta, V^s) = 0$.

Similarly matched workers in state (V, z) solve

$$\max_{V^s, \theta}\left\{p\left(\theta\right)V^s + \left(1 - p\left(\theta\right)\right)V\right\}$$

subject to $J^m(\theta, V^s, V, z) = 0$.

Once a worker transits to a firm offering an expected value V^s, match quality is revealed. Let $\Pi(V, z)$ denotes the maximum expected present value of profits for a firm that has promised V to a worker when match quality is z. Firms chooses contingent continuation values V_H and V_L conditional on the realization of match quality, subject to a promise-keeping constraint. Thus,

$$E\left[\Pi\left(V^s\right)\right] = \max_{V_H, V_L}\left\{\mu_H \Pi\left(V_H, z_H\right) + \left(1 - \mu_H\right)\Pi\left(V_L, z_L\right)\right\} \tag{1}$$

subject to

$$\mu_H V_H + \left(1 - \mu_H\right)V_L \geq V^s.$$

We next describe the firm dynamic profit maximization, for a firm with a promise V

and known match quality z. We will think of the firm as directing the choices of the worker about when to quit and about where to direct on the job search, recognizing that those choices must be consistent with utility maximization by the worker. The firm chooses a current period wage w, and a promised continuation value V' if the match survives into the next period and if worker does not report an outside offer after the search and matching phase at the start of the next period. The firm also chooses the promised value $V^{s'}$ and tightness $\theta^{'}$ of the market to which the worker will direct job search in the next period. None of these choices are contingent on the realization of the worker's disutility of work shock χ. It follows that a worker will choose to quit when his realization of the preference shock exceeds a threshold $\bar{\chi}$ defined by

$$U(w) - \bar{\chi} + \beta p(\theta^{'})V^{s\prime} + \beta \left(1 - p(\theta^{'})\right) V' = V^u \tag{2}$$

The left hand side of this expression defines the worker's expected present value when they do not quit: they receive the wage w, pay the utility cost of work, and enter the next period matched. In the next period, the worker receives an outside offer with probability $p(\theta^{'})$ in which case their continuation value is $V^{s\prime}$, while with reciprocal probability their continuation value is V'.[6] If they quit, the worker simply enjoys the present value of not working in the current period, which we denote V^u. Note that the firm's choices of w, V', $V^{s\prime}$ and θ' will influence the value for $\bar{\chi}$ that leaves the worker indifferent about quitting or staying with the firm.

The contract offered by the firm must deliver V to the worker. This promise-keeping constraint can be written as

$$\gamma F(\bar{\chi}) \left[U(w) - E\left[\chi | \chi \leq \bar{\chi}\right] + \beta p(\theta^{'})V^{s\prime} + \beta \left(1 - p(\theta^{'})\right) V'\right] + (1 - \gamma F(\bar{\chi}))V^u \geq V \tag{3}$$

Here $\gamma F(\bar{\chi})$ is the probability the worker produces at the firm in the current period, which happens if they neither separate exogenously nor quit. The term in square brackets is expected lifetime utility in that scenario. With reciprocal probability, the worker separates

[6] Note that if the worker does receive an outside offer, the workers continuation value is $V^{s\prime}$ independent of whether the existing firm matches the offer and retains the worker or does not match, in which case the worker moves to the new firm.

in the current period, and receives V^u.

Next, the choices for $V^{s\prime}$ and θ' must be consistent with optimal job search behavior on the part of the worker and those choices must be consistent with zero profits for firms posting in the market

$$\left(V^{s\prime},\theta'\right) \in \arg\max \left\{ p\left(\theta'\right) V^{s\prime} + \left(1 - p\left(\theta'\right)\right) V' \right\} \tag{4}$$

subject to

$$q\left(\theta'\right)(1-\bar{\zeta}\left(V^{s\prime},V',z\right))E\left[\Pi\left(V^{s\prime}\right)\right] - \phi = 0 \tag{5}$$

Finally, the firm's choice for the probability of matching outside offers ζ' must be such that a worker without an outside offer weakly prefers not to falsely report an offer:

$$\zeta' V^{s\prime} + (1-\zeta')V^u \leq V', \tag{6}$$

where the left hand side is the payoff to a worker without an outside offer who falsely reports having an offer, while the right hand side is the payoff if the worker truthfully reports no outside offer.[7] Note that if the worker reports having an outside offer, the incumbent firm will infer that the value of that offer is the value $V^{s\prime}$ that solves the worker's on the job search problem.

Thus, the firm's dynamic optimization problem is

$$\Pi(V,z) = \max_{\{w,\bar{\chi},V',\theta',V^{s\prime},\zeta'\}} \gamma F(\bar{\chi}) \left[zY - w + \beta\left(1-p(\theta')\right)\Pi(V',z) + \beta p(\theta')\zeta'\Pi(V^{s\prime},z)\right] \tag{7}$$

subject to constraints given by eqs. 2, 3, 4, 5 and 6. Note that flow current period profits are given by $zY - w$, but these are only realized if the match is not destroyed and the worker does not quit. Expected continuation profits vary depending on whether or not the worker receives an outside offer at the start of the next period, and, if they do, whether or not the firm matches the offer.

[7] Note that the offer matching probabilities in eq: 5 are not chosen by the firm, but taken as given.

2.1 Definition of Equilibrium

A stationary equilibrium is a set of values for unemployed workers, $\{V_n^u\}$ (one for each sector n), search choices for unemployed workers $\left\{\theta_n^u, V_{0,n}^u\right\}$, decision rules $\{V_H(V), V_L(V), w_n(V,z), \bar{\chi}_n(V,z), V_n'(V,z), V_n^{s\prime}(V,z), \theta_n'(V,z), \zeta_n'(V,z)\}$ for all (V,z) and for all n, profit values $\{\Pi_n(V,z), E\left[\Pi_n\left(V^s\right)\right]\}$, and offer matching probability functions $\bar{\zeta}_n\left(V^s, V, z\right)$ s.t.

1. Given V_n^u and $(V_n'(V,z), V_n^{s\prime}(V,z), \theta_n'(V,z))$, the decision rule for quitting thresholds $\bar{\chi}_n(V,z)$ satisfies the worker optimal quitting condition eq. 2 for all (V,z).

2. The decision rule for offer matching $\zeta_n'(V,z)$ satisfies eq. 6 with equality when $\Pi_n(V_n^{s\prime}(V,z),z) \geq 0$ and is zero otherwise (firms retain profitable matches as often as possible while preserving truthtelling).

3. The job matching probabilities that posting firms take as given are consistent with optimal offer matching: $\bar{\zeta}_n\left(V^{s\prime}, V', z\right) = \zeta_n'(V,z)$ when $V^{s\prime} = V_n^{s\prime}(V,z)$ and $V' = V_n'(V,z)$.

4. Job search choices $(V_n^{s\prime}(V,z), \theta_n'(V,z))$ are welfare-maximizing for workers subject to eq. 5, taking as given offer matching probabilities $\bar{\zeta}_n\left(V^{s\prime}, V', z\right)$ and expected profits $E\left[\Pi_n\left(V^{s\prime}\right)\right]$.

5. Given V_n^u, $\bar{\chi}_n(V,z)$, $\bar{\zeta}_n\left(V^{s\prime}, V', z\right)$, and $\left(V_n^{s\prime}(V,z), \theta_n'(V,z)\right)$, the decision rules $(w_n(V,z), \bar{\chi}_n(V,z), V_n'(V,z))$ satisfy the firm's profit maximization problem for all (V,z) and corresponding profits are given by $\Pi_n(V,z)$.

6. Given $\Pi_n(V,z_H)$ and $\Pi_n(V,z_L)$ the decision rules $V_H(V)$ and $V_L(V)$ solve problem 1 and $E\left[\Pi_n\left(V^s\right)\right]$ is the associated expected profit value for all V^s.

7. $\left(\theta_n^*, V_{0,n}^*\right)$ maximizes welfare for unmatched workers within set of values (θ_n, V_n^s) that satisfy $q\left(\theta_n\right) E\left[\Pi_n\left(V_n^s\right)\right] - \phi_n = 0$.

8. Unmatched values satisfy $V_n^u = U(b(n)) + \beta\left(p(\theta_n^*)V_{0,n}^* + (1 - p(\theta_n^*))V_n^u\right)$.

9. Revenue from taxes at rate τ finances benefits to unmatched workers.

10. In each sector, the measure of unmatched workers and the joint distribution of workers over states (V, z) is constant over time.

3 Theoretical Characterization

In this section we characterize aspects of the model equilibrium focusing on two questions. First, given the private information friction, what is the form of optimal contractual arrangements between firms and workers? Second, given the market response to the private information friction, how should the government design labor market policies, in particular transfer payments to the nonemployed?

To answer the first question, we focus on two important aspects of private contracting design in a directed search framework. First, at the search and matching stage, workers trade off higher (average) wages offered versus a lower job finding probability. The reason is that in markets offering higher promised values (expected wages), firms expect lower profits and, as a result, optimally post fewer vacancies, implying a lower job finding probability. Second, after the search and matching stage, the firm designs a dynamic wage schedule that delivers the promised value. Two insights are obtained.

First, given the quitting friction, optimal dynamic contracts must feature backloading of wage payments. Second, at the search and matching stage, workers direct search to markets with relatively high promised values (and lower job finding probabilities). We identify this an an "efficiency wage" channel – the market converges on a higher wage equilibrium because higher wages reduce quitting. While this high wage, low job finding equilibrium is privately optimal, the equilibrium vacancy rate may be too low relative to the social optimum, and equilibrium non-employment may be too high.

Having characterized the private contracting arrangement, we proceed to study the optimal design of unemployment insurance. The insight here is that the unemployment insurance should be less generous when the private information friction is present. We deliver this insight in a simplified static model with linear utility, in which we can solve the model in closed-form, and compare the solution to classic results in Acemoglu and Shimer (1999).

3.1 Backloading of Wages

We start by characterizing the firm's dynamic optimization problem (7). To focus on the quitting margin due to preference shocks – the novel aspect of the model – we assume no on-the-job search in this section. This simplified contracting problem is:

$$\Pi(V,z) = \max_{w,V',\bar{\chi}} \left\{ \gamma F(\bar{\chi})(zY - w + \beta\Pi(V',z)) \right\} \tag{8}$$

$$U(w) - \bar{\chi} + \beta V' = V^u$$

$$\gamma F(\bar{\chi}) \left(U(w) - E\left[\chi | \chi \leq \bar{\chi}\right] + \beta V' \right) + (1 - \gamma F(\bar{\chi}))V^u \geq V$$

where the first constraint states that the quitting threshold $\bar{\chi}$ satisfies the worker's indifference condition (2), while the second constraint is the promise-keeping condition (3).

In this problem, the firm seeks to deliver the promised value V in the most profitable way. Given that worker utility is concave, it is conceivable that the firm would like to smooth wages over time. But is a perfectly flat wage profile optimal? The answer is no. Because of the quitting friction, the firm wants to backload wages, because by doing so it can reduce the worker's future incentives to quit, and thereby increase the discounted value of the match. The following inverse Euler equation characterizes the solution to the firm problem:

Proposition 3.1 *Wage growth under the contract that solves the firm problem (8) satisfies:*

$$\frac{1}{U'(w_{t+1})} - \frac{1}{U'(w_t)} = \frac{f(\bar{\chi}_{t+1})}{F(\bar{\chi}_{t+1})} \left[zY - w_{t+1} + \beta\Pi_{t+2} \right] \tag{9}$$

The wage rises for as long as the worker and firm remain matched, and converges to zY.

Condition (9) can be understood in the following way. Imagine that the firm needs to deliver some promised value V_t to a worker, and is contemplating some wage sequence that delivers that value.

Now consider the perturbation of reallocating one unit of worker value from period t to period $t+1$. The left hand side of the equation measures the net pecuniary cost to the firm, where the firm's profit is increased by $\frac{1}{U'(w_t)}$ in period t and reduced by $\frac{1}{U'(w_{t+1})}$ in period $t+1$. The right hand side measures the benefit of increasing future promised value: a higher future value V_{t+1} raises the $t+1$ quitting threshold $\bar{\chi}_{t+1}$, leading to reduced quitting in proportion to the normalized density function $\frac{f(\bar{\chi}_{t+1})}{F(\bar{\chi}_{t+1})}$. The value of the match is then increased by the change in the probability of quitting times the future expected profit if the worker remains with the firm, $zY - w_{t+1} + \beta\Pi_{t+2}$. Under the optimal wage sequence, the cost of an additional delay to worker compensation must equate the benefit, resulting in (9).

As long as the match remains profitable, the right hand side of condition (9) will be positive, and thus the left hand side must be too. Given concave utility, this implies that w_{t+1} must be greater than w_t, i.e., wages are backloaded. A very simple way to understand this backloading result is that a higher wage at t reduces quitting at t but not at $t+1$, while a higher promised wage at $t+1$ reduces quitting at both t *and* $t+1$. As wage growth continues, wages converge to productivity, zY.

3.2 Outside Offer Matching

The model features a probabilistic offer matching rule that disincentivizes workers from faking receipt of an outside offer (equation 6). In equilibrium, workers search for offers that yield higher value than their current match: $V^{s\prime} \geq V'$. Whether the incumbent firm will consider matching depends on whether the match would still be profitable at the higher promised value. If $\Pi(V^{s\prime}, z) \geq 0$, then the firm will want to match and will choose the highest incentive-compatible matching probability:

$$\zeta' = \frac{V' - V^u}{V^{s\prime} - V^u}.$$

The matching probability is decreasing in $V^{s\prime}$ because a higher $V^{s\prime}$ makes mimicking more attractive to workers who do not have an outside offer. Thus, the firm needs to impose a harsher punishment (a higher probability of firing instead of matching) to deter such mimicking. When $V^{s\prime}$ is sufficiently high, the value of the match to the incumbent firm drops below zero: $\Pi(V^{s\prime}, z) < 0$. In this case the offer matching rate is zero.

Proposition 3.2 *The offer matching function $\bar{\zeta}\left(V^{s\prime}, V^{\prime}, z\right)$ is given by:*

$$\bar{\zeta}\left(V^{s\prime}, V^{\prime}, z\right)=\begin{cases}\frac{V^{\prime}-V^{u}}{V^{s\prime}-V^{u}} & \text{if } \Pi(V^{s\prime}, z) \geq 0 \\ 0 & \text{otherwise}\end{cases}$$

3.3 Optimal Unemployment Insurance: A Closed Form

Having characterized the firm's dynamic contracting problem, we now move to the search and matching stage. We consider first a tractable, static version of the model. Analytic tractability will make it easy to understand how the quitting margin impacts the directed search equilibrium and how it impacts optimal policy design. In the next section we will revisit the analysis in the richer dynamic version of the model.

In the static version of the model there is a single sector, and there is no uncertainty about match quality: all matches produce z. A continuum of workers are initially unmatched, and their utility functions are assumed linear. Vacancies can be created at cost ϕ. As in the benchmark model, once matched, workers are subject to a stochastic preference shock χ drawn from a cumulative distribution function $F(.)$.

In this environment, we first define a first-best allocation. Suppose a benevolent social planner maximizes the ex-ante welfare of all unmatched workers. The planner cannot bypass the search friction but can choose how many vacancies to create and, upon observing the χ shocks, can also dictate the workers' quitting decisions, conditional on their idiosyncratic χ shocks. Given linear utility, welfare maximization boils down to maximizing social net output: aggregate output minus the utility costs of working and the costs of posting vacancies:

$$\max_{\theta, \bar{\chi}} p(\theta) \int^{\bar{\chi}} (z-\chi)\, dF-\theta \phi \tag{10}$$

where θ denotes market tightness and, given a unit mass of initially unmatched workers, also equals the number of vacancies posted.

Under the first best allocation, since the planner can observe the χ shock, all matches with positive surplus will be preserved. In other words, a match is preserved if and only if its productivity is greater than the labor disutility cost, and hence the quitting threshold under the first-best allocation is exactly equal to productivity:

$$\bar{\chi} = z.$$

Given the first-best threshold $\bar{\chi}$, the optimal θ can be found by taking a first order condition with respect to the objective function (10).

We now move to the competitive equilibrium. In order to isolate the role of the information friction, we consider two scenarios. In the first, the χ shock is publicly observable and hence wage contracts can be made contingent on χ. In the second scenario, the χ shock is not observable, and thus wage contracts must be independent of the χ shock. We assume that the government runs a balanced budget by imposing a lump-sum tax τ on all workers and paying a transfer b to all unmatched workers. Hence a worker's utility with offered wage w and realized preference shock χ is given by

$$U^e = w - \tau - \chi$$

while a non-employed worker receives

$$U^n = b.$$

We assume that the matching function is Cobb-Douglas with share parameter equal to 0.5. This implies that the worker's job finding probability is

$$p = A\sqrt{\theta}.$$

The preference shock is drawn from a uniform distribution with support $[0, a]$. With these assumptions on the matching function and the preference shock distribution, we can characterize the competitive equilibrium in closed-form.

Proposition 3.3 *Given policy parameters b and τ, competitive equilibrium allocations in the economies in which χ is publicly observable and in which χ is not observable are given by*

	public χ		private χ
$\bar{\chi}$	$z - (\tau + b)$	$>$	$\frac{3}{4}z - \frac{3}{4}(\tau + b)$
$E[w]$	$\frac{3}{4}z + \frac{1}{4}(\tau + b)$	$=$	$\frac{3}{4}z + \frac{1}{4}(\tau + b)$
p	$\frac{A^2}{\phi}\frac{1}{a}\frac{1}{4}(z - (\tau + b))^2$	$>$	$\frac{A^2}{\phi}\frac{3}{4a}\frac{1}{4}(z - (\tau + b))^2$

where $E[w]$ denotes the expected wage, conditional on $\chi < \bar{\chi}$.

Hence, introducing the private information friction leads to

1. *A lower quitting threshold $\bar{\chi}$ and a higher quit rate,*
2. *No change in the average wage paid, and*
3. *A lower equilibrium job finding probability.*

This proposition highlights the efficiency wage channel via which the market addresses excess quitting. In the public χ economy, the firm can reduce quitting without raising the average wage paid by promising higher wages when the realization of χ is high, and lower wages when it is low. Spreading out wages in this fashion is not costly to the worker given linear utility. In the private χ economy, in contrast, the firm can only pay a single wage, and the only way to reduce quitting is to raise that wage. Thus, conditional on any given value for the expected wage $E[w]$, there is more quitting in the private χ economy, meaning that vacancy postings are less profitable for the firm. The firm responds by posting fewer vacancies, translating to a less favorable tradeoff for the worker at the search and matching stage, with a lower expected wage for any given job finding probability.

This tradeoff is depicted in Figure 3.[8] The solid lines are the set of feasible $(E(w), p)$ pairs derived from the firms' free entry conditions. One can think of them as the budget sets faced by the workers. The dashed lines are indifference curves from the worker's perspective. The points where the two curves are tangent are the competitive equilibria. The blue lines depict the public χ economy while the red lines show the private χ economy.

[8]The parameter values used to construct the plot are: $A = 1.5$, $a = 2$, $z = 1$, $\phi = 0.5$, $b = 0$.

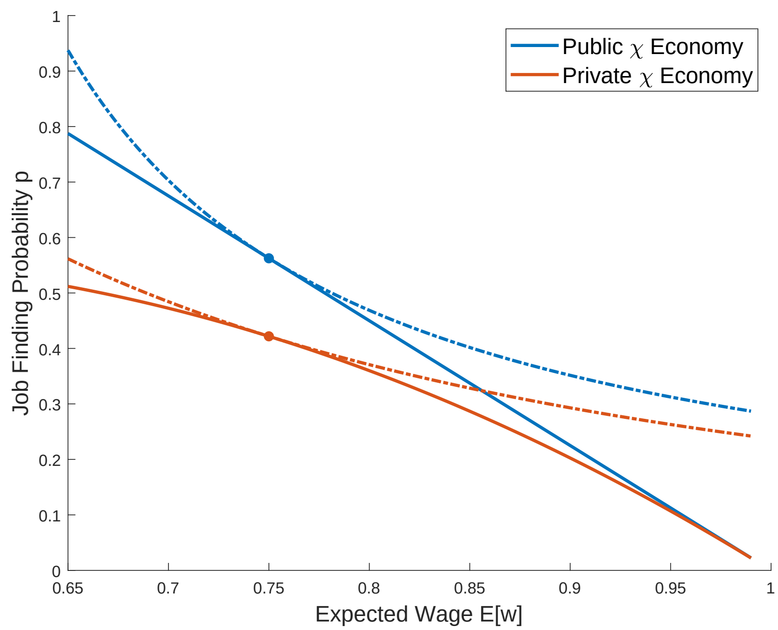

Figure 3: Equilibria in the Public and Private χ Economies. The solid lines show the set of sub-markets available to searching workers. The dashed lines are indifference curves. The dots are the equilibrium choices.

Note that the budget line for the private χ economy in Figure 3 lies below the one for the public χ economy, indicating that the quitting threshold shrinks the worker's budget set. But more importantly, the budget line is also flatter in the private χ economy. The reason is that, comparing across labor sub-markets offering different $(E[w], p)$ combinations, the quitting threshold in the public χ economy is always equal to $\bar{\chi} = z - (\tau + b)$ and is thus independent of the expected wage, while the threshold in the private χ economy is given by $\bar{\chi} = w - (\tau + b)$ and is thus increasing in w, implying a quit rate that is declining in w. It follows that firms in the private χ economy lose fewer profits due to quitting in relatively high wage markets. Thus, in the private χ economy firms are willing to post higher wages in exchange for only a small increase in their working finding probability q, because part of the direct cost of paying a higher wage is offset by a smaller chance that the match dissolves. From the worker standpoint, that makes searching in a relatively high wage market more attractive.

Thus, in the economy with private information, workers and firms coordinate on a relatively high wage equilibrium. This outcome is consistent with the longstanding notion of "efficiency wages." One way to interpret this outcome is that even though workers cannot commit not to quit *ex ante*, workers can alleviate the quitting friction by choosing to direct search to a high-wage market, thereby making it more costly for them to quit *ex post*.

We now explore optimal policy. The government chooses the transfer to non-employed workers b to maximize worker expected welfare, subject to a balanced government budget constraint:

$$pF(\bar{\chi})\,\tau = (1 - pF(\bar{\chi}))\,b.$$

Our results on optimal policy are summarized in the following proposition:[9]

Proposition 3.4 *In the public χ economy, the optimal unemployment benefit b^* and tax rate τ^* are 0, while in the private χ economy, b^* and τ^* are negative:*

	public χ	private χ
$(\tau^* + b^*)$	0	$-\frac{z}{5}$

In the public χ economy, the optimal policy delivers the first-best allocation. In the private χ economy, welfare under the optimal policy is strictly less than under the first-best allocation.

The proposition can be understood as follows. There are two behavioral margins where workers' choices might not be socially optimal, and where there is a potential role for policy to change those choices. One is the search margin: the planner would like workers to search in the "right" submarket in which the firms are posting the optimal number of vacancies. The second margin is the quitting margin, which determines how many profitable matches are preserved ex-post: the planner would like to preserve all matches where $\chi \leq z$.

In the public χ economy, private markets can deliver efficient quitting behavior via $\chi-$contingent wage payments, and thus there is no need for policy intervention to address that margin. Moving to the search margin, absent policy intervention, directed search delivers the socially efficient level of market tightness and vacancy posting. The

[9] We summarize policy by the optimal value for $b + \tau$, since policy parameters enter all equilibrium variables in the form of this sum (see the previous proposition). It is straightforward to compute the optimal value for b given $b + \tau$ using the equilibrium expressions for $\bar{\chi}$ and p and the government budget constraint.

intuition is straightforward: workers simply choose to search in the submarket that offers the highest expected welfare, subject to firms making zero profits. Acemoglu and Shimer (1999) consider a directed search environment similar to ours, and also find that a zero unemployment benefit restores the first best allocation. Hence, in the public χ economy, a combination of $b = 0$ and a state contingent wage contract replicates the first best allocation. Note that allocations would not be efficient for $b \neq 0$: for example, with $b > 0$ workers would quit too often.

In the private χ economy, the firm loses the ability to offer wage payments conditional on χ, which is unobservable. As a result, the quit rate is higher than the first best when $b = 0$. In addition, the previous proposition indicates that given $b = \tau = 0$, job searchers are too picky relative to the first best, because of the efficiency wage channel discussed above. Thus, too few vacancies are posted relative to the first best.

Reducing the unemployment benefit to $b < 0$ helps to alleviate distortions on both margins. With a less generous unemployment benefit and hence a worse outside option, matched workers choose to quit less often. At the same time, job seekers become less picky ex-ante and direct search toward markets with more vacancies, pushing the competitive allocation closer to the first-best. However, since the government only has one independent policy instrument $b$it cannot, in general, deliver the first best choice on both margins.[10] The optimal value $\tau^* + b^* = -\frac{z}{5}$ strikes a balance, delivering $\bar{\chi} = 0.9z$ (so there is still too much quitting) but a job finding rate p that is above the first best value (so there is also too much vacancy creation).[11]

[10] The government can also decide on τ, but it needs to maintain a balanced budget, so it really only has only one independent policy lever.

[11] An additional potential instrument is a wage policy. One can show that a combination of a minimum wage policy and less generous UI can replicate the first best. Intuitively a minimum wage policy ensures that the right amount of vacancies are posted in equilibrium, while the UI benefit is set so that there is the right amount of quitting.

4 Quantitative Results

4.1 Calibration

The model period length is set to one month. We assume workers have logarithmic utility from consumption, and set $\beta = 0.96$ on an annual basis. We assume a Cobb-Douglas matching technology, with productivity A, such that if v vacancies are posted and u workers search, the number of meetings is $m = A\sqrt{uv}$. The preference shock χ is assumed to be drawn from a Lognormal distribution, with mean μ_χ and variance σ_χ^2 : $\chi \sim LN(\mu_\chi, \sigma_\chi^2)$. We assume that the cost of posting vacancies is proportional to average earnings in the sector: $\phi_n = \hat{\phi} Y_n$. The average quality of new matches is normalized to 1: $\mu_H z_H + (1 - \mu_L) z_L = 1$.

We identify different model sectors with the sectors for which data on job openings, layoffs and quits are available in the CPS JOLTS database, and for which data on employment and average weekly earnings are also available in the CES survey.[12] Most of our empirical targets are averages for the US private sector over the 12 months from July 2021 to June 2022, over which period we think of the US economy as being in steady state.[13] For each of our sectors, we define the share of the population in sector n, λ_n, as employment in that sector relative to total employment across all sectors. And we define the productivity of sector n, Y_n, as average weekly earnings in that sector relative to average earnings across all sectors. Thus, $\sum_{n=1}^{N} \lambda_n Y_n = 1$. [14]

The unemployment benefit system is assumed to take the following form

$$b(n) = \delta + \kappa \min\{Y_n, 1\}. \tag{11}$$

The parameter δ captures a floor on benefits, while the parameter κ captures the com-

[12] These sectors are: mining and logging, construction, durable manufacturing, non-durable manufacturing, wholesale trade, retail trade, transportation, warehousing and utilities, information, financial activities, professional and business services, education and health, arts, entertainment and recreation, accommodation and food services, other services.

[13] The Covid pandemic era expansions of UI benefits ended on September 6, 2021.

[14] In the equilibrium of our model, individual earnings are endogenous, and vary within sectors with idiosyncratic match quality, tenure, and the history of outside offers. But average equilibrium earnings within a sector n turn out to be very close to the exogenous sectoral productivity parameter Y_n.

ponent of benefits that is proportional to earnings, up to a cap.[15] We will set $\delta = 0.05$ and $\kappa = 0.5$. The logic for these choices is as follows. The two most important sources of benefits for unemployed Americans are unemployment insurance and SNAP (food stamps) (see, e.g., Bitler et al. 2020, or Elsby et al. 2022). While the generosity of unemployment benefits varies significantly across different US states, many states offer replacement rates around 50 percent, up to a cap of around \$500 per week, which is about half of average weekly earnings. That is our rationale for setting $\kappa = 0.5$. SNAP benefits are worth around \$250 per month (not per week) per recipient household suggesting a modest floor δ on benefits. Note that in our model, all non-workers are eligible for benefits, including those who quit. In reality, some non-workers, especially those who quit their previous job, may be ineligible for unemployment insurance. In Section 4.3 we will explore optimal transfer policy when the government is allowed to differentiate payments between non-workers who quit versus those who were fired.

This leaves seven parameters: the exogenous match destruction rate $1 - \gamma$, the vacancy cost and match efficiency parameters $\hat{\phi}$ and A, the mean and variance of the disutility parameter μ_χ and σ_χ^2, and the ratio of high to low match quality and the share of high quality matches, z_H/z_L and μ_H. Three of our targets for these parameters come from JOLTS. We set γ to match the sum of the JOLTS layoff rate plus the JOLTS "other separations" rate (other separtions includes retirements, deaths and disability), which is $1.66 + 0.28 = 1.94$ percent per month.[16] We also target the JOLTS job opening rate $(8.03$ percent per month$)$ and the JOLTS quit rate $(3.69$ percent$)$. One more target is the CPS unemployment rate, which was 4.15 percent for the private sector. Loosely speaking, the job opening rate and the unemployment rate identify the vacancy posting cost $\hat{\phi}$ and match efficiency parameter, A, while the quit rate identifies the average disutility cost of work, μ_χ.

An important target for us is the share of quits that involve transitions to a period of non-employment, versus quits that are associated with an immediate transfer to another job. For this, we turn to the Census Job to Job Explorer, which is based on LEHD data.

[15] Our assumption that benefits are linked to average sectoral earnings rather than to individual earnings simplifies the analysis, because it implies that all non-workers in a sector have the same expected value V^u.

[16] We incorporate the adjustments proposed by Davis et al. (2010) to the published JOLTS numbers, and scale up the published layoff, other separation, job openings, and quit rates by factors of 1.64, 1.29, 1.08 and 1.15, respectively, following their Table 5.4.

Averaging over quarterly data from 2000 Q2 to 2021 Q2, 32.1 percent of all separations are job-to-job separations with continuous employment. Applied to our total JOLTS separation rate of $1.94 + 3.69 = 5.63$ percent, that suggests a monthly job-to-job quit rate of $0.321 \times 5.63 = 1.81$ percent. Many parameters affect the model job-to-job transition rate, but an important one is the share of matches that are initially good, μ_H : if drawing a good match is less likely, there is more equilibrium churn as workers repeatedly sample new firms and quit low quality matches. Given a total quit rate 3.69 percent, the targetted quit to non-employment rate is $3.69 - 1.81 = 1.88$ percent.[17]

To identify the productivity differential between good and bad matches, z_H/z_L, we target average wage growth associated with job to job transitions. Birinci et al. (2022) estimate 9 percent growth in earnings for continuously employed workers upon a job change using the LEHD.

One parameter remains, which is the variance of idiosyncratic preference shocks, σ_χ^2. This parameter is important because it determines how sensitive the model quit rate is to policy parameters. We set σ_χ^2 to replicate the observed elasticity of the sectoral quit rate to sectoral average earnings. Intuitively, if the quit rate is much higher in high wage sectors (like finance) relative to low wage sectors (like accommodation and food services) that would point to a relatively low value for σ_χ^2, while if quits are largely uncorrelated with wages, that would point to a high value. Figure 4.1 plots sectoral quit rates against sectoral average wages, and indicates that quits are indeed heavily concentrated in low wage sectors.

All parameter values are reported in Table 1.

[17] Fujita et al. (2022) estimate a job-to-job transition rate from the CPS, and show that a change in the CPS survey methodology can partly explain a puzzling decline in the measured CPS J2J rate. Their proposed correction gives a J2J rate of around 2 percent, similar to our target. However, they highlight two important open data puzzles. One is that, even with their correction, the CPS-measured total quit rate looks pretty flat over the post Global Financial Crisis period, while there is a dramatic upward trend in the JOLTS quit rate. A second puzzle is that the level of quits in the CPS is much higher than in JOLTS. Both these puzzles are clearly illustrated in Figure 11 of their Online Appendix.

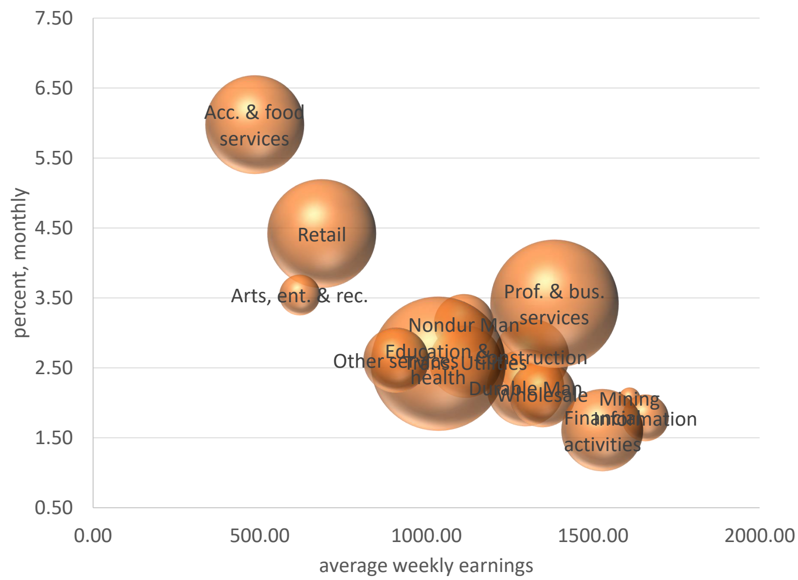

Figure 4: JOLTS Quit Rates by Industry, 2021-22. The bubble sizes reflect industry employment shares.

Table 1: Calibration

Parameters		Value	Source/Targets
Externally Calibrated			
Discount factor	β	$0.99^{1/3}$	Monthly model
Non-employed consumption	δ, κ	0.05,0.5	SNAP+UI
Exogenous separation rate	$1-\gamma$	1.94%	JOLTS layoffs + other separations
Sector weights/earnings	$\{\lambda_n, Y_n\}$		CES
Internally Estimated			
Labor disutility shocks, mean	μ_χ	-0.95	Quit to non-employment rate 1.88%
Labor disutility shocks, variance	σ_χ^2	0.25	Elasticity of quits to sectoral earnings
Vacancy posting cost	ϕ	0.165	Job opening rate 8.03%
Match quality dispersion	z_H/z_L	1.14	Wage growth for job switchers 9%
Share of high quality matches	μ	0.5	Job-to-job transition rate 1.81%
Matching efficiency	A	0.75	Unemployment rate 4.15%

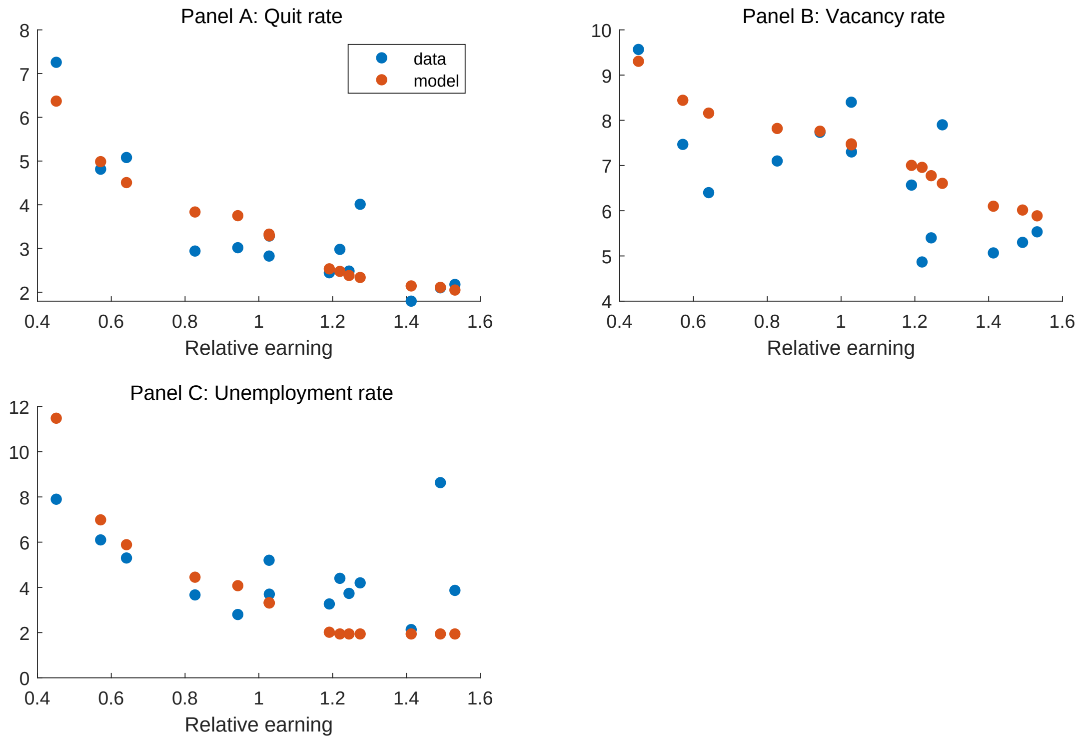

Figure 5: Quit Rate, Vacancy Rate, and Unemployment Rate by Sector: Model versus Data

4.2 Quantitative Results

Figure 5 plots various industry-level labor market statistics generated by the model (red dots), against those in the data (blue dots). Given our calibration strategy, the model matches quite well the variation in quit rates across different industries (Panel A). In terms of untargetted moments, the model also performs reasonably with respect to cross-industry variation in job opening rates and unemployment rates (Panels B and C respectively), both of which tend to be higher in low wage industries.

Quits in the model can be decomposed into quits to non-employment and quits to another job. Figure 6 plots such a decomposition across different industries, where the blue dots show the share of workers who quit into non-employment while the red dots show job-to-job transitions. The negative relationship in the model between industry earnings and the industry quit rate is entirely driven by variation in the quit-to-non-employment rate; the job-to-job transition rate varies little with sectoral earnings.[18] The fact that the

[18]Bosler and Petrosky-Nadeau (2016) document that the job-to-job transition rate is similar across different

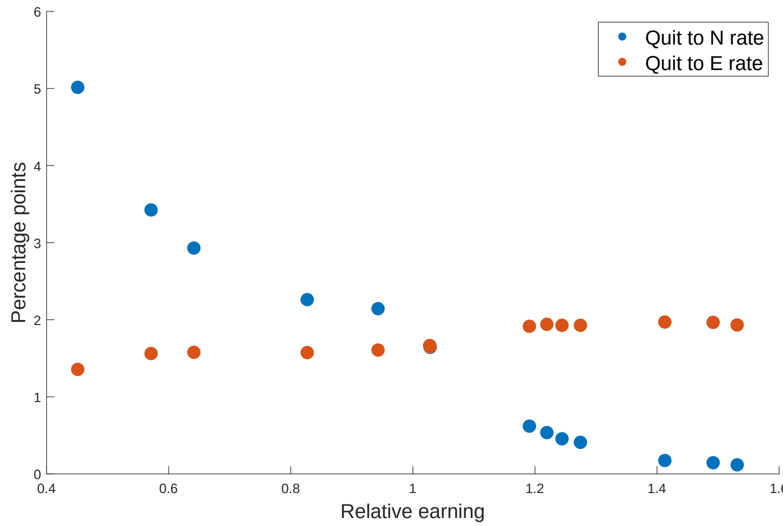

Figure 6: Two Types of Quits in the Model

quit to non-employment rate is quite sensitive to sector-level earnings suggests a preference shock distribution with significant density in the region where low wage workers will optimally quit but high wage workers will not.[19]

The compressed labor disutility shock distribution has important consequences for our policy analysis. It means that reducing unemployment insurance can have a large impact on worker's quit rates, and hence can lead to large welfare improvements.

Figure 7 plots various value and policy functions conditional on whether the worker is in a high-quality (left column) or low-quality (right column) match, in a high productivity sector ($Y_n = 1.5$). The first row plots continuation values $V'(V)$ as a function of current promised value (red line). For high-quality matches, workers start with low promised value, but experience growth in promised value and wages (second row, left panel) with tenure. Workers with short tenure and low current promised values engage in on the job search (third row, left panel). If they succeed in finding an outside offer, their incumbent firm matches the offer with a high probability (fourth row), so only a few such workers

occupations among recent cohorts of young workers.

[19]There is an analogy here to the unemployment volatility puzzle (Shimer, 2005). The resolution proposed by Hagedorn and Manovskii (2008) is that the value of unemployment is high, so that workers are close to indifferent between working or not. This indifference generates high unemployment sensitivity with respect to aggregate labor productivity shocks. In our context, we observe a high sensitivity of the quit rate to sectoral earnings in cross-section, suggesting that the mean of the labor disutility shock must be high, and its variance must be low.

switch employers. Lastly, the worker's quit to non-employment rate decreases as the worker's promised value increases.[20]

Turning to low quality matches, there is very little wage growth for incumbents and workers search for outside offers as a way to boost their income (yellow line, top right panel). Workers choose to search in submarkets where their job finding probability is around 50 percent (third row, right panel). The incumbent firm does not match those offers, given that their value exceeds the maximum value of the current low-quality match. Finally, the quit to non-employment rate is much higher for workers in low quality matches compared to those in high quality matches (last row).

Figure 8 shows how wages rise with tenure and how the quit rate declines with decline. High quality matches feature more scope for wage growth while in low quality matches workers are paid their labor productivity starting from the second month of employment. This is due to an insurance mechanism: recall that firms need to allocate promised values conditional on the realization of match quality (equation 1). Given concave utility, the firm optimally cross subsidizes workers who realize low match quality, so their initial wage is close to their marginal product. But the firm does not perfectly insure match quality risk, because offering more to workers in high quality matches reduces the quit rate for those profitable matches.

To close this section, Figure 9 plots the sample path for a worker who starts out unemployed (red dots). She finds a job at month two, and the match turns out to be low quality (represented by blue dots). At month four she quits the job due to a high χ preference shock. After several months of unemployment she finds another job, which this time turns out to be a high quality match (green dots). This lasts for quite a while, and wages rise with tenure. Then the match ends exogenously. The next job she finds is low quality, but through on-the-job search she eventually transitions to a high-quality match.

[20]Interestingly, the model only features offer matching for workers in high wage sectors. The reason is that for high wage workers, the benefit replacement rate is low. Thus, unmatched workers in high wage sectors direct search to sub-markets with high job finding probabilities, but with low promised values. Because their initial equilibrium promised values are low (relative to expected productivity,) it is profitable for other firms to try to poach recent hires.

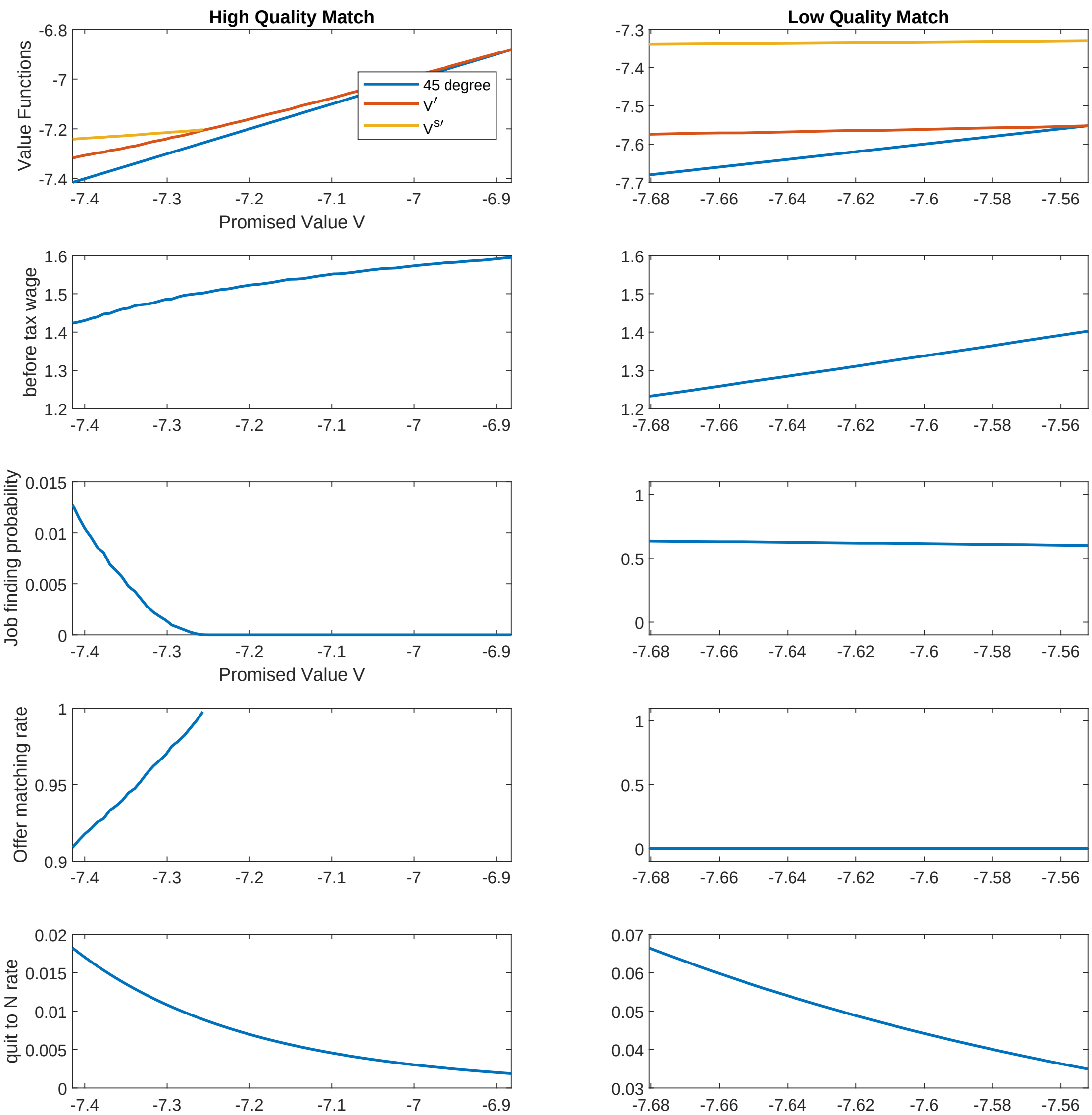

Figure 7: Value and Policy Functions. The left-side panels are for workers in a high quality match. The right-side panels are for workers in a low quality match. The x axis for each figure shows current promised value V. The x axes ranges show the set of values for V observed in equilibrium, conditional on match quality.

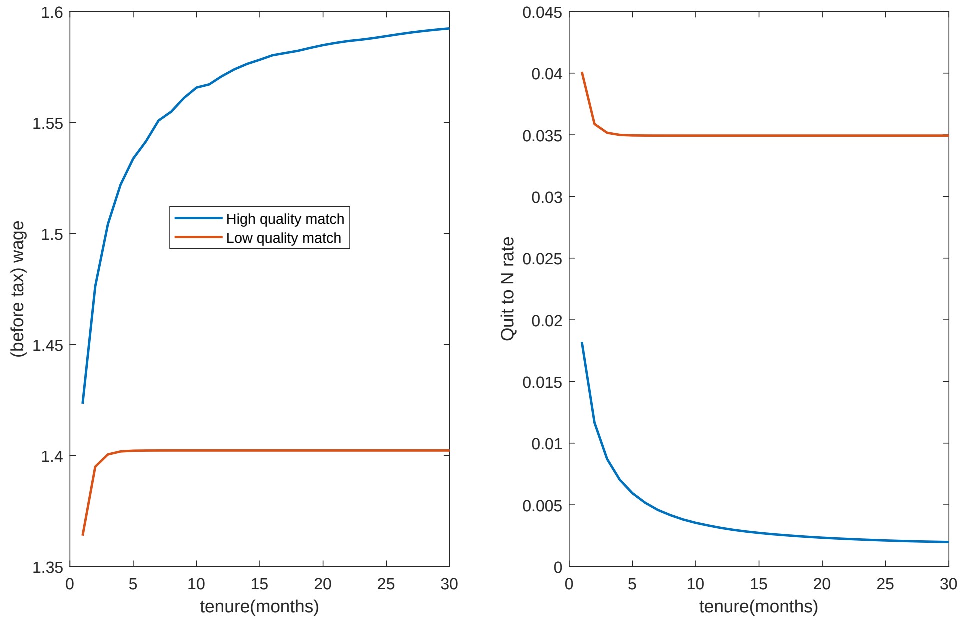

Figure 8: Wages and Quit Rate by Tenure (worker in sector with $Y_n = 1.5$)

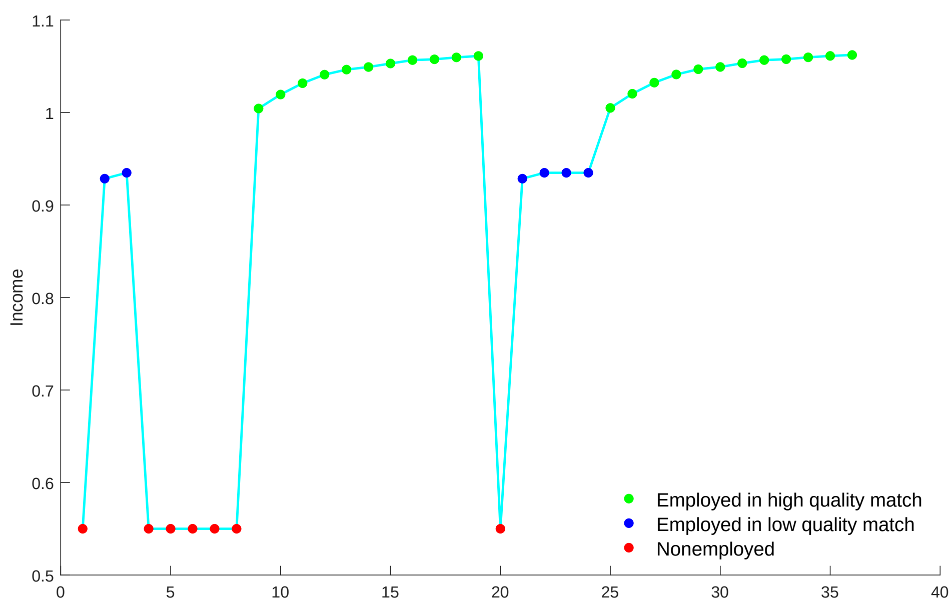

Figure 9: Sample Path for Disposable Income (worker in sector with $Y_n = 1$)

Table 2: Optimal Replacement Rates

	US Policy	Optimal Policy
κ^* (%)	50.0	38.4
EN rate (%)	1.80	0.46
EE rate (%)	1.85	2.09
u rate (%)	4.13	1.98
v rate (%)	7.69	6.82
p rate (%)	78.1	98.7

4.3 Optimal Policy

We turn now to optimal policy. We assume that benefits to non-employed workers are always specified by the function described in equation 11 and optimize with respect to the parameter κ. We maximize with respect to the average (across sectors) expected value for a non-working individual in steady state, $\sum_{n=1}^{N} \lambda_n V_n^u$.[21]

The welfare maximizing policy involves reducing κ from 50.0 percent to 38.4 percent. Table 2 shows how this changes key labor market statistics. Optimally reducing the replacement rate cuts the quit to non-employment rate from 1.8 to 0.5 percent. Non-employed workers also become less picky, directing search to markets where they almost surely find jobs immediately. The combined impact of these changes is that the unemployment rate is cut in half.

How important is the presence of the quitting margin in driving our baseline policy prescription? To explore this we now consider several counter-factuals; see Table 3. In the first, we shrink the variance σ_χ^2 of the idiosyncratic preference shock to near zero (while adjusting the mean parameter μ_χ so as to leave the expected value for χ unchanged). In this scenario (column 2 of the table) the optimal replacement rate is 48.9 percent, which is 10.5 percentage points higher than the optimum in the baseline model and very similar to our calibrated value.

At first sight, the optimal replacement rate in this specification might still seem rather low: becoming unemployed still leads to a 50 percent decline in consumption. The only

[21] We have also considered an objective of average lifetime utility in cross section; results were very similar. We do not incorporate transitional dynamics in our policy analysis, but transitions are known to be quick in this class of models.

behavioral margin sensitive to the UI replacement rate in this specification is job pickiness, and at the optimum replacement rate, 87.5 percent of unemployed workers find a job within a month. The reason the planner does not want to push the replacement rate higher is that the job finding rate is sensitive to κ, and would decline to zero if pushed much higher. In our calibrations, the expected utility cost of working is $E[\chi] = \exp(\mu_\chi + \sigma_\chi^2/2) = 0.439$. Given that cost, a household is indifferent between working or not when $\log(1-\tau) - 0.439 = \log(\kappa)$. That calculation suggests a maximum possible replacement rate of around 63.8 percent.[22] If the planner were to try to push the replacement rate above that threshold, no-one would choose to work.

In the next experiment, we switch off quits to other jobs, by assuming that working individuals cannot search for new jobs. In this case, the optimal replacement rate is 44.0 percent, 5.6 percentage points higher than in the baseline version of the model. That might seem surprisingly high, given that the quit-to-nonemployment margin is still active here, and that is the margin that is directly sensitive to benefit generosity. But note that when on-the-job search is ruled out, the only way workers can escape from low quality matches is to quit to non-employment, and then search. The lower are benefits, the less inclined workers will be pursue this route, so the planner increases benefits relative to the baseline to preserve incentives to transition out of bad matches.

Note that in our model, both EE and EN transitions can be inefficient in that workers leave firms when the value to the firm of remaining matched is positive. But there are two reasons why quits to non-employment tend to be more costly, from a social welfare perspective. The first is that the model features a fiscal externality: if an individual quits to non-employment, workers collectively must pay higher taxes to finance the benefits that individual will receive. Because workers do not internalize this, they tend to quit to non-employment too often.[23] This fiscal externality is absent in job-to-job transitions. The second reason job-to-job transitions are less costly is that, in our calibration, some of the

[22]For this calculation, we assume $\tau = 0.011$, which is the budget balancing rate at the optimum. But if the planner were to raise the replacement rate, the budget-balancing tax rate would rise, in part because a lower p would imply more benefit recipients. Thus, the maximum feasible replacement rate is in fact lower than 63.8 percent.

[23]Note that this fiscal externality was absent in the simple example with linear utility and full information we considered earlier, because benefits were optimally zero in that case.

Table 3: Optimal Policies under Counter-factuals

	Optimal Policies			
	Baseline	$\sigma_\chi^2 = 0.01$	No OJS	$\frac{z_H}{z_L} = 1$
κ^* (%)	38.4	48.9	44.0	33.5
EN rate (%)	0.46	0.07	1.42	1.08
EE rate (%)	2.09	1.78	0.00	0.04
u rate (%)	1.98	2.32	2.38	1.95
v rate (%)	6.82	5.19	7.42	5.43
p rate (%)	98.7	87.5	92.7	99.7

quits to non-employment are from high quality matches, while all of the job-to-job quits are from low quality matches. Thus on-the-job search is a better technology for improving match quality.

In our next experiment, we eliminate variation in match quality. Now, there is no efficiency rationale for job to job transitions – all jobs are equally productive. We find an even lower optimal replacement rate in this case: 33.5 percent. Our interpretation is that in the baseline model (with match quality risk), the planner is less concerned about quitting, because most quitters are workers in bad matches. The surplus from these matches is small, so the planner is less concerned about them being destroyed. In addition, workers who quit transition to better matches at a faster rate than those who remain employed: under our calibration to the United States, only 50 percent of employed workers in bad matches transition to a new match each month via on-the-job search (Figure 7) compared to 78 of non-employed workers. Thus, in the baseline specification, quitting speeds up reallocation to better matches. When we switch off variation in match quality, quitting becomes more costly and this beneficial effect of quitting on reallocation is no longer operative. Thus, the optimal replacement rate is lower.

In our last policy experiment, we assume the government can perfectly identify two different types of non-workers: those who voluntarily quit from their previous job, versus those who experienced an exogenous separation shock. We allow the government to pay different benefits to those who quit versus those who were fired. In the United States, the government does try to separately identify these two groups, and the former is usually ineligible for unemployment benefits.

Table 4 reports the results from this experiment. The planner chooses different replacement rates for the two groups, with the firees getting 48.5 of earnings replacement (up to the cap) while the quitters get only 29.8 percent. The planner gives the quitters less to reduce excessive quitting. Note, however, that it is not optimal to give the quitters nothing: workers who choose to quit do so because they face a high cost of working, and the planner does not want to enforce zero quitting by threatening quitters with starvation. Note also that the optimal replacement rate for firees remains below 50 percent (as it is in the experiment in which we effectively eliminate quitting) so workers who separate exogenously still face large declines in consumption.

Suppose the planner can verify whether a non-employed individual quit or was fired, but that it must pay to acquire this information. How large a cost is worth paying? To answer this question we compare the welfare gains of two different policy reforms. The first is our baseline reform (Table 2) in which we move from a uniform 50 percent replacement rate, to a uniform 38.4 percent replacement rate. This reform raises welfare for a non-working firee by an amount equivalent to a permanent 1.0 percent in increase in consumption. The second reform is replacing a uniform 38.4 percent replacement rate with the type-specific replacement rates for quitters versus firees reported in the last column of Table 4. The corresponding welfare gain from this reform is much smaller, at 0.3 percent of consumption. Whether or not it is worth paying the cost of ascertaining whether or not workers quit or fired will depend on whether one can do so at a cost of less than 0.3 percent of aggregate consumption.

4.4 Explaining The Great Resignation

The quit rate and the vacancy rate are both much higher at the end of our sample period than they were at the end of the previous expansion in 2006 (see Figures 1 and 2). It is difficult to separate cycle from trend over the relatively short sample period for which JOLTS data are available, but the figures suggest a secular upward trend in both series. We now use the model to offer a tentative explanation for observed changes in labor market dynamics. In particular, we should that a decline in vacancy posting costs can rational-

Table 4: Optimal Policy with Differential Benefits

	Actual	Optimal Policies	
		Baseline	$\kappa^*_{EU} \neq \kappa^*_{EN}$
κ^*_{EU} (%)	50.0	38.4	48.5
κ^*_{EN} (%)	50.0	38.4	29.8
EN rate (%)	1.80	0.46	0.01
EE rate (%)	1.85	2.09	1.97
u rate (%)	4.13	1.98	2.26
v rate (%)	7.69	6.82	5.26
p_U rate (%)	78.1	98.7	87.5
p_N rate (%)	78.1	98.7	100.0

Table 5: The Great Resignation Via Lower Vacancy Costs

	2006	2021-22	Δ (pp)	Δ Model
EN rate (%)	0.8	1.8	1.0	0.9
EE rate (%)	1.8	1.8	0.0	0.3
u rate (%)	4.6	4.1	-0.5	-1.0
v rate (%)	4.0	7.7	3.7	3.5

ize most of the key dynamics observed over this period. One interpretation is that new technologies and platforms such as Monster and Indeed have made it cheaper for firms to keep maintain active search for new workers.

Table 5 reports the results from the following experiment. We start from our model calibration to 2021-2022. Relative to that calibration we change only one parameter, which is the vacancy posting cost parameter $\hat{\phi}$. We search for the value $\hat{\phi}_{2006}$ so that, in steady state, the model comes as close as possible to matching the 2006 values for the unemployment rate, the job openings rate, the EN transition rate, and the EE transition rate.[24] The resulting estimate for $\hat{\phi}_{2006}$ is 0.320, almost twice the value the 2021/22, which is 0.165. Comparing across steady states, moving from $\hat{\phi}_{2006}$ to $\hat{\phi}_{2021-22}$ increases the model quit-to-non-employment rate by 0.9 percentage points, while increasing the job-openings rate by 3.5 percentage points. Both changes are very similar to those observed in the data. The model predicts a small increase in the job-to-job transition rate, while there was no such increase in the data.

5 Conclusion

The message of the paper is that the quitting margin plays a very important quantitative role in optimal unemployment insurance design, and benefits are optimally much less generous in a setting with a quitting margin. There are two reasons why. The first is that when a worker initiates a separation, the remaining surplus to the firm from preserving the match is positive, almost by definition. Thus quitting destroys value. Firms can try to reduce quitting by backloading wages and by stochastically matching outside offers. But to the extent that quits are driven by private idiosyncratic preference shocks, there will still be too much quitting in equilibrium. Making benefits to non-workers less generous reduces the number of inefficient quits.

The second reason why benefits should be lower with a quitting margin is that one way private markets try to address the quitting problem is by co-ordinating on a high

[24]Our data targets are 12 month averages of these statistics for 2006. Our distance metric is the sum of squared deviations between model and data moments.

"efficiency" wage equilibrium. Workers search for high wage jobs, because firms expect less quitting in such jobs, reducing the full cost of paying higher wages. Thus, in our directed search setting workers pay a relatively small penalty in terms of longer expected job search time in exchange for a higher expected wage. But while efficiency wages are privately welfare maximizing, this partial fix to the quitting problem exacerbates the fiscal externality associated with UI benefits: workers do not internalize that the longer they spend searching for high efficiency wage jobs, the higher must be equilibrium tax rates to finance benefits.

Our model could be extended in various dimensions. One would be to introduce persistent preference shocks in order to generate persistent periods of non-employment. Another would be to add shocks to match productivity during the entire duration of a match, instead of only at the time the match is formed, as a way to generate a richer cross-sectional distribution of wages. A third would be to introduce savings and potentially other sources of self-insurance against unemployment risk. Such extensions might change our quantitative policy prescriptions, but the basic mechanisms we have identified would remain.

References

Acemoglu, Daron and Robert Shimer, "Efficient Unemployment Insurance," *Journal of Political Economy*, 1999, *107* (5), 893–928.

Baily, Martin Neil, "Some Aspects of Optimal Unemployment Insurance," *Journal of Public Economics*, 1978, *10* (3), 379–402.

Balke, Neele and Thibaut Lamadon, "Productivity Shocks, Long-Term Contracts, and Earnings Dynamics," *American Economic Review*, July 2022, *112* (7), 2139–77.

Birinci, Serdar, Fatih Karahan, Yusuf Mercan, and Kurt See, "Labor Market Shocks and Monetary Policy," *Working paper*, 2022.

Bitler, Marianne, Hilary W Hoynes, and Diane Whitmore Schanzenbach, "The Social Safety Net in the Wake of COVID-19," September 2020, (27796).

Blanco, Julio, Andres Drenik, Christian Moser, and Emilio Zaratiegui, "A Theory of Non-Coasean Labor Markets," 2023.

Bosler, Canyon and Nicolas Petrosky-Nadeau, "Job-to-Job Transitions in an Evolving Labor Market," *FRBSF Economic Letter*, 2016, p. 34.

Burdett, Ken and Melvyn Coles, "Equilibrium Wage-Tenure Contracts," *Econometrica*, 2003, *71* (5), 1377–1404.

Chetty, Raj, "A General Formula for the Optimal Level of Social Insurance," *Journal of Public Economics*, 2006, *90* (10), 1879–1901.

Davis, Steven J., R. Jason Faberman, John C. Haltiwanger, and Ian Rucker, "Adjusted Estimates of Worker Flows and Job Openings in JOLTS," in "Labor in the New Economy" NBER Chapters, National Bureau of Economic Research, Inc, 2010, pp. 187–216.

Delacroix, Alain and Shouyong Shi, "Directed Search on the Job and the Wage Ladder," *International Economic Review*, 2006, *47* (2), 651–699.

Elsby, Michael, Axel Gottfries, Ryan Michaels, and David Ratner, "Vacancy Chains," Working Papers 22-23, Federal Reserve Bank of Philadelphia August 2022.

Elsby, Michael W L and Axel Gottfries, "Firm Dynamics, On-the-Job Search, and Labor Market Fluctuations," *The Review of Economic Studies*, 09 2021, *89* (3), 1370–1419.

Fujita, Shigeru, Giuseppe Moscarini, and Fabien Postel-Vinay, "Measuring Employer-to-Employer Reallocation," *American Economic Journal: Macroeconomics*, July 2022.

Ganong, Peter, Fiona Greig, Pascal Noel, Daniel M. Sullivan, and Joseph Vavra, "Spending and Job Search Impacts of Expanded Unemployment Benefits: Evidence from Administrative Micro Data," *Working paper*, 2022.

Golosov, Mikhail, Pricila Maziero, and Guido Menzio, "Taxation and Redistribution of Residual Income Inequality," *Journal of Political Economy*, 2013, *121* (6), 1160–1204.

Hagedorn, Marcus and Iourii Manovskii, "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited," *American Economic Review*, September 2008, *98* (4), 1692–1706.

Johnston, Andrew C. and Alexandre Mas, "Potential Unemployment Insurance Duration and Labor Supply: The Individual and Market-Level Response to a Benefit Cut," *Journal of Political Economy*, 2018, *126* (6), 2480–2522.

Karahan, Fatih, Kurt Mitman, and Brendan Moore, "Micro and Macro Effects of UI Policies: Evidence from Missouri," *Working paper*, 2022.

Menzio, Guido and Shouyong Shi, "Efficient Search on the Job and the Business Cycle," *Journal of Political Economy*, 2011, *119* (3), 468–510.

Mercan, Yusuf and Benjamin Schoefer, "Jobs and Matches: Quits, Replacement Hiring, and Vacancy Chains," *American Economic Review: Insights*, March 2020, *2* (1), 101–24.

Moore, John, "Optimal Labour Contracts when Workers have a Variety of Privately Observed Reservation Wages," *The Review of Economic Studies*, 1985, *52* (1), 37–67.

Postel–Vinay, Fabien and Jean–Marc Robin, "Equilibrium Wage Dispersion with Worker and Employer Heterogeneity," *Econometrica*, 2002, *70* (6), 2295–2350.

Qiu, Xincheng, "Vacant Jobs," *Working paper*, 2022.

Shi, Shouyong, "Directed Search for Equilibrium Wage–Tenure Contracts," *Econometrica*, 2009, *77* (2), 561–584.

Shimer, Robert, "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, March 2005, *95* (1), 25–49.

_ , "On-the-Job Search and Strategic Bargaining," *European Economic Review*, 2006, *50* (4), 811–830.

Stevens, Margaret, "Wage-Tenure Contracts in a Frictional Labour Market: Firms' Strategies for Recruitment and Retention," *The Review of Economic Studies*, 04 2004, *71* (2), 535–551.

Appendix

A Proofs

Proof of Proposition 3.1

$$\Pi(V) = \max_{w,V'} \left\{ \gamma F(\bar{\chi}(w,V'))(z - w + \beta \Pi(V')) \right\}$$

s.t.

$$\log(w) - \bar{\chi}(w,V') + \beta V' = V^u : \textit{quitting\_rule}$$

$$\gamma F(\bar{\chi}(w,V')) \left(U(w) - E\left[\chi | \chi \leq \bar{\chi}(w,V')\right] + \beta V' \right) + (1 - \gamma F(\bar{\chi}(w,V')))V^u \geq V : \textit{promise\_keeping}$$

The quitting rule gives a closed form for $\bar{\chi}(w,V')$

We can plug that into the second constraint and into the objective

So we have an objective with choice variables w and V' and a constraint with the same two

So we have

$$\bar{\chi}(w,V') = U(w) + \beta V' - V^u$$

So

$$\Pi(V) = \max_{w,V'} \left\{ \gamma F(U(w) + \beta V' - V^u)(z - w + \beta \Pi(V')) \right\}$$

s.t.

$$\gamma F(U(w) + \beta V' - V^u) \left(U(w) - E\left[\chi | \chi \leq U(w) + \beta V' - V^u\right] + \beta V' \right) + (1 - \gamma F(U(w) + \beta V' - V^u))V^u = V : \mu$$

Where we denote the multiplier μ.

The promise keeping constraint can be rewritten as:

$$\gamma F(U(w) + \beta V' - V^u) \left(U(w) + \beta V' \right) - \gamma \int_{-\infty}^{U(w)+\beta V' - V^u} \chi f(\chi)\, d\chi + (1 - \gamma F(U(w) + \beta V' - V^u))V^u = V : \mu$$

The Lagrangian is given by

$$
\begin{aligned}
L &= \gamma F(U(w)+\beta V'-V^u)(z-w+\beta\Pi(V')) \\
&+\mu\left(V-\gamma F(U(w)+\beta V'-V^u)\left(U(w)+\beta V'\right)+\gamma\int_{-\infty}^{U(w)+\beta V'-V^u}\chi f\left(\chi\right)d\chi-(1-\gamma F(U(w)+\beta V'-V^u))V^u\right)
\end{aligned}
$$

Take FOCs

$$
\begin{aligned}
w &: -\gamma F(U(w)+\beta V'-V^u)+\gamma f(U(w)+\beta V'-V^u)(z-w+\beta\Pi(V'))U'(w) \\
&-\mu\begin{pmatrix}\gamma F(U(w)+\beta V'-V^u)U'(w)+\gamma f(U(w)+\beta V'-V^u)\left(U(w)+\beta V'\right)U'(w) \\ -\gamma\left(U(w)+\beta V'-V^u\right)f\left(U(w)+\beta V'-V^u\right)U'(w) \\ -\gamma f\left(U(w)+\beta V'-V^u\right)V^uU'(w)\end{pmatrix} \\
&= 0
\end{aligned}
$$

divide through by $f(.), U'(w)$, and γ :

$$
\begin{aligned}
w &: -\frac{F(U(w)+\beta V'-V^u)}{f(U(w)+\beta V'-V^u)}\frac{1}{U'\left(w\right)}+(z-w+\beta\Pi(V')) \\
&-\mu\begin{pmatrix}\frac{F(U(w))+\beta V'-V^u)}{f(U(w)+\beta V'-V^u)}+\gamma\left(U(w)+\beta V'\right) \\ -\left(U(w)+\beta V'-V^u\right) \\ -V^u\end{pmatrix} \\
&= 0
\end{aligned}
$$

$$
w:-\frac{F(U(w)+\beta V'-V^u)}{f(U(w)+\beta V'-V^u)}\frac{1}{U'\left(w\right)}+z-w+\beta\Pi(V')-\mu\frac{F(U(w)+\beta V'-V^u)}{f(U(w)+\beta V'-V^u)}=0
$$

Rearrangment w have the following relation:

$$
w:\frac{f\left(\bar{\chi}\right)}{F\left(\bar{\chi}\right)}\left[z-w+\beta\Pi(V')\right]=\mu+\frac{1}{U'\left(w\right)}
$$

The other first order condition w.r.t V' is:

$$\begin{aligned} V' \quad &: \quad \beta\gamma f(U(w)+\beta V'-V^u)(z-w+\beta\Pi(V'))+\beta\gamma F(U(w)+\beta V'-V^u)\Pi'(V') \\ & -\mu\left(\begin{array}{c} \beta\gamma f(U(w)+\beta V'-V^u)\left(U(w)+\beta V'\right)+\beta\gamma F(U(w)+\beta V'-V^u) \\ -\beta\gamma\left(U(w)+\beta V'-V^u\right) f\left(U(w)+\beta V'-V^u\right) \\ -\beta\gamma f(U(w)+\beta V'-V^u))V^u \end{array}\right) \\ &= \quad 0 \end{aligned}$$

Divide through with $\beta\gamma f\,(.)$:

$$\begin{aligned} V' \quad &: \quad (z-w+\beta\Pi(V'))+\frac{F}{f}\Pi'(V') \\ & -\mu\frac{F}{f} \\ &= \quad 0 \end{aligned}$$

Thus we have

$$\frac{f\left(\bar{\chi}\right)}{F\left(\bar{\chi}\right)}\left[z-w+\beta\Pi(V')\right]=\mu-\Pi'(V')$$

Combining the two we have

$$\frac{1}{U'\left(w_t\right)}=-\Pi'(V_{t+1})$$

Now we need to obtain an expression for $\Pi'(V_{t+1})$, take the envelope theorem:

$$\Pi'(V_{t+1})=\mu_{t+1}=\frac{f\left(\bar{\chi}_{t+1}\right)}{F\left(\bar{\chi}_{t+1}\right)}\left[zY-w_{t+1}+\beta\Pi(V_{t+1})\right]-\frac{1}{U'\left(w_{t+1}\right)}$$

Hence we have

$$\frac{1}{U'\left(w_{t+1}\right)}-\frac{1}{U'\left(w_t\right)}=\frac{f\left(\bar{\chi}_{t+1}\right)}{F\left(\bar{\chi}_{t+1}\right)}\left[zY-w_{t+1}+\beta\Pi(V_{t+1})\right]$$

Proof of Proposition 3.3

We start by solving the equilibrium in the public χ economy. The problem is given by the following:

$$\max_{\theta,w,\bar{\chi}} p\left(\theta\right)\int^{\bar{\chi}}\left(w-\tau-\chi\right)dF\left(\chi\right)+\left[1-p\left(\theta\right)F(\bar{\chi})\right]b$$

$$s.t.$$

$$q(\theta)\int^{\bar{\chi}}(z-w)\,dF(\chi)=\phi$$

$$\bar{\chi}=z-\tau-b$$

Plug in $\bar{\chi}=z-\tau-b$:

$$\max_{\theta,w,\bar{\chi}} p(\theta)\int^{z-\tau-b}(w-\tau-\chi)\,dF(\chi)+[1-p(\theta)\,F(z-\tau-b)]\,b$$

$$s.t.$$

$$q(\theta)\int^{z-\tau-b}(z-w)\,dF(\chi)=\phi$$

With the assumption that χ is uniformly distributed, we have:

$$\max_{\theta,w} p(\theta)\frac{\left[(w-\tau)(z-\tau-b)-\frac{1}{2}(z-\tau-b)^2\right]}{a}+b-p(\theta)\frac{z-\tau-b}{a}b$$

$$s.t.$$

$$q(\theta)\frac{z-\tau-b}{a}(z-w)=\phi$$

Collect terms the objective function becomes:

$$p(\theta)\left[\frac{(z-\tau-b)(w-\tau-b)}{a}-\frac{(z-\tau-b)^2}{2a}\right]+b$$

With the assumptions on the matching function

$$p(\theta)=A\theta^{0.5}$$

$$q(\theta)=A\theta^{-0.5}$$

Substitute out θ :

$$q(p)=\frac{A^2}{p}$$

We can transform this problem into

$$\max_{w,p} p\left[\frac{(z-\tau-b)(w-\tau-b)}{a}-\frac{(z-\tau-b)^2}{2a}\right]+b$$

$$s.t.$$

$$\frac{A^2}{p}\frac{z-\tau-b}{a}(z-w)=\phi$$

The constraint says:

$$p=\frac{A^2}{\phi}\frac{z-\tau-b}{a}(z-w)$$

Plug this into the objective:

$$\max_{w}\frac{A^2}{\phi}\frac{z-\tau-b}{a}(z-w)\left[\frac{(z-\tau-b)(w-\tau-b)}{a}-\frac{(z-\tau-b)^2}{2a}\right]+b$$

Take first order condition with respect to w :

$$-\left[\frac{(z-\tau-b)(w-\tau-b)}{a}-\frac{(z-\tau-b)^2}{2a}\right]+(z-w)\frac{(z-\tau-b)}{a}=0$$

$$-(w-\tau-b)+\frac{(z-\tau-b)}{2}+z-w=0$$

$$w=\frac{3}{4}z+\frac{1}{4}(\tau+b)$$

With wage given by the above expression, job finding probability p is given by

$$\begin{aligned}p &= \frac{A^2}{\phi}\frac{z-\tau-b}{a}(z-w)\\ &= \frac{A^2}{\phi}\frac{z-\tau-b}{a}\left(z-\left(\frac{3}{4}z+\frac{1}{4}(\tau+b)\right)\right)\\ &= \frac{A^2}{\phi}\frac{(z-\tau-b)^2}{4a}\end{aligned}$$

The case of public χ is fully characterized. We now move to the case with private χ.

The problem is given by

$$\max_{\theta,w,\bar{\chi}} p(\theta) \int^{\bar{\chi}} (w-\tau-\chi)\, dF(\chi) + [1-p(\theta) F(\bar{\chi})]\, b$$

s.t.

$$q(\theta) \int^{\bar{\chi}} (z-w)\, dF(\chi) = \phi$$

$$w-\tau-\bar{\chi} = b$$

Substitute in $\bar{\chi} = w-\tau-b$:

$$\max_{\theta,w,\bar{\chi}} p(\theta) \int^{w-\tau-b} (w-\tau-\chi)\, dF(\chi) + [1-p(\theta) F(w-\tau-b)]\, b$$

s.t.

$$q(\theta) \int^{w-\tau-b} (z-w)\, dF(\chi) = \phi$$

Plug in the uniform distribution of χ :

$$\max p(\theta) \left[(w-\tau-b) \frac{(w-\tau-b)}{a} - \frac{1}{2a} (w-\tau-b)^2 \right] + b$$

s.t.

$$q(\theta) \frac{w-\tau-b}{a} (z-w) = \phi$$

Plug in

$$q(p) = \frac{A^2}{p}$$

We have

$$\max p \left[(w-\tau-b) \frac{(w-\tau-b)}{a} - \frac{1}{2a} (w-\tau-b)^2 \right] + b$$

$$\frac{A^2}{p} \frac{w-\tau-b}{a} (z-w) = \phi$$

Or

$$p = \frac{A^2}{\phi} \frac{w-\tau-b}{a} (z-w)$$

Plug it into the objective:

$$\max \frac{A^2}{\phi}\frac{w-\tau-b}{a}\left(z-w\right)\left[\left(w-\tau-b\right)\frac{\left(w-\tau-b\right)}{a}-\frac{1}{2a}\left(w-\tau-b\right)^2\right]+b$$

$$\max \frac{A^2}{\phi}\frac{\left(w-\tau-b\right)^3}{a}\left(z-w\right)\frac{1}{2a}+b$$

Take FOC with respect to w :

$$3\left(w-\tau-b\right)^2\left(z-w\right)+-\left(w-\tau-b\right)^3=0$$

$$3z-3w-w+\tau+b=0$$

$$w=\frac{3}{4}z+\frac{1}{4}(\tau+b)$$

Hence the quitting threshold is given by

$$\begin{aligned}\bar{\chi} &= w-\tau-b \\ &= \frac{3}{4}z+\frac{1}{4}(\tau+b)-\tau-b \\ &= \frac{3}{4}\left(z-\tau-b\right)\end{aligned}$$

And the job finding probability is given by

$$\begin{aligned}p &= \frac{A^2}{\phi}\frac{w-\tau-b}{a}\left(z-w\right) \\ &= \frac{A^2}{\phi}\frac{3}{4a}\frac{1}{4}\left(z-\tau-b\right)^2\end{aligned}$$

Proof of Proposition 3.4

We first derive the indirect value function for the workers, we then show that there exists a closed form in both the public χ and the private χ economy.

For the public χ economy, denote this value function $V(\tau, b)$:

$$\begin{aligned} V(\tau, b) &= p\int^{z-\tau-b} [w-\tau-\chi]\, dF(\chi) + (1-pF(z-\tau-b))\, b \\ &= pF(z-\tau-b)(w-\tau) - p\int^{z-\tau-b} \chi dF(\chi) + (1-pF(z-\tau-b))\, b \\ &= pF(z-\tau-b)(w-\tau-b) + b - \frac{p}{2a}(z-\tau-b)^2 \end{aligned}$$

Using the government budet constraint:

$$pF(z-\tau-b)\,\tau = (1-pF(z-\tau-b))\, b$$

we have

$$pF(z-\tau-b)(b+\tau) = b$$

Using this relation to substitute out the middle term b, we have:

$$\begin{aligned} V(\tau, b) &= pF(z-\tau-b)(w-\tau-b) + pF(z-\tau-b)(b+\tau) - \frac{p}{2a}(z-\tau-b)^2 \\ &= pF(z-\tau-b)\, w - \frac{p}{2a}(z-\tau-b)^2 \\ &= p\frac{z-\tau-b}{a} w - \frac{p}{2a}(z-\tau-b)^2 \end{aligned}$$

Plug in the expression for w and p :

$$w = \frac{3}{4}z + \frac{1}{4}(\tau+b)$$

$$p = \frac{A^2}{\phi}\frac{(z-\tau-b)^2}{4a}$$

We have

$$\begin{aligned}
V(\tau, b) &= p\frac{z-\tau-b}{a}w - \frac{p}{2a}(z-\tau-b)^2 \\
&= \frac{A^2}{\phi}\frac{(z-\tau-b)^3}{4a^2}\left[\frac{3}{4}z + \frac{1}{4}(\tau+b) - \frac{2}{4}(z-\tau-b)\right] \\
&= \frac{A^2}{\phi}\frac{(z-\tau-b)^3}{16a^2}[z-\tau-b] \\
&= \frac{A^2}{\phi}\frac{(z-\tau-b)^4}{16a^2}
\end{aligned}$$

It is easy to see that V is minimized when $\tau + b = 0$. By the budget constraint if $\tau + b = 0$ it follows that b must also be zero. Hence the Acemoglu-Shimer result holds in this environment. We need to show that this coincides with the first-best allocation.

Under the first best allocation, the planner solves:

$$\max_{\theta, \bar{\chi}} p(\theta) \int^{\bar{\chi}} (z-\chi)\, dF(\chi) - \theta\phi$$

Plug in the uniform distribution and $p(\theta) = A\theta^{\frac{1}{2}}$, and substitue in $\bar{\chi} = z$:

$$\max_{\theta} A\theta^{\frac{1}{2}} \frac{\frac{1}{2}z^2}{a} - \theta\phi$$

Take FOC:

$$\frac{1}{2}A\theta^{-\frac{1}{2}}\frac{\frac{1}{2}z^2}{a} = \phi$$

$$\frac{1}{4}A\frac{z^2}{a\phi} = \theta^{\frac{1}{2}}$$

$$\theta = \left(\frac{1}{4}A\frac{z^2}{a\phi}\right)^2$$

Plug it back into the objective. The value becomes:

$$A\frac{1}{4}A\frac{z^2}{a\phi}\frac{\frac{1}{2}z^2}{a} - \left(\frac{1}{4}A\frac{z^2}{a\phi}\right)^2\phi$$

$$
\begin{aligned}
& \frac{1}{8}A^2\frac{z^4}{a^2\phi}-\frac{1}{16}A^2\frac{z^4}{a^2\phi} \\
= & \frac{1}{16}A^2\frac{z^4}{a^2\phi}
\end{aligned}
$$

Which is exactly equal to the value in the public χ economy. Hence we confirm that the first best allocation can be achieved in the public χ economy.

Let's move to the private χ economy. Denote this value function $V(\tau,b)$:

$$
\begin{aligned}
V(\tau,b) &= p\int^{w-\tau-b}[w-\tau-\chi]\,dF(\chi)+(1-pF(w-\tau-b))\,b \\
&= pF(w-\tau-b)(w-\tau)-p\int^{w-\tau-b}\chi dF(\chi)+(1-pF(w-\tau-b))\,b \\
&= pF(w-\tau-b)(w-\tau-b)+b-\frac{p}{2a}(w-\tau-b)^2
\end{aligned}
$$

Using the government budet constraint:

$$
pF(w-\tau-b)\,\tau=(1-pF(w-\tau-b))\,b
$$

We have

$$
\begin{aligned}
V(\tau,b) &= pF(w-\tau-b)(w-\tau-b)+pF(w-\tau-b)(b+\tau)-\frac{p}{2a}(w-\tau-b)^2 \\
&= pF(w-\tau-b)\,w-\frac{p}{2a}(w-\tau-b)^2 \\
&= p\frac{w-\tau-b}{a}w-\frac{p}{2a}(w-\tau-b)^2
\end{aligned}
$$

Plug in

$$
w=\frac{3}{4}z+\frac{1}{4}(\tau+b)
$$

$$
p=\frac{A^2}{\phi}\frac{3}{4a}\frac{1}{4}(z-\tau-b)^2
$$

we have

$$
\begin{aligned}
V(\tau, b) &= p\frac{(w-\tau-b)}{a}\left[w-\frac{w-\tau-b}{2}\right] \\
&= p\frac{(w-\tau-b)}{a}\left[\frac{w+\tau+b}{2}\right] \\
&= \frac{A^2}{\phi}\frac{3}{4a}\frac{1}{4}(z-\tau-b)^2\frac{\left(\frac{3}{4}z+\frac{1}{4}(\tau+b)-\tau-b\right)}{a}\left[\frac{\frac{3}{4}z+\frac{1}{4}(\tau+b)+\tau+b}{2}\right] \\
&= \frac{A^2}{\phi}\frac{3}{4a}\frac{1}{4}(z-\tau-b)^2\frac{\left(\frac{3}{4}z-\frac{3}{4}(\tau+b)\right)}{a}\left[\frac{\frac{3}{4}z+\frac{5}{4}(\tau+b)}{2}\right] \\
&= \frac{A^2}{\phi}\frac{3}{4a}\frac{1}{4}\frac{3}{4}\frac{(z-(\tau+b))^3}{a}\left[\frac{\frac{3}{4}z+\frac{5}{4}(\tau+b)}{2}\right]
\end{aligned}
$$

Take derivative with respect to

$$x = \tau + b$$

we have:

$$-3(z-x)^2(3z+5x)+5(z-x)^3 = 0$$

$$-3(3z+5x)+5(z-x) = 0$$

$$5z-5x = 9z+15x$$

$$20x = -4z$$

$$x = -\frac{z}{5}$$

Hence the optimal policy in this case calls for taxing the unemployed:

$$\tau + b = -\frac{z}{5}$$

With the optimal policy, the value function is given by

$$
\begin{aligned}
V(\tau, b) &= \frac{A^2}{\phi}\frac{3}{4a}\frac{1}{4}\frac{3}{4}\frac{\left(z+\frac{z}{5}\right)^3}{a}\left[\frac{\frac{3}{4}z-\frac{5}{4}\frac{z}{5}}{2}\right] \\
&= \frac{A^2}{\phi}\frac{3}{4a}\frac{1}{4}\frac{3}{4}\left(\frac{6}{5}\right)^3\frac{1}{4}\frac{z^3}{a}z \\
&= \frac{A^2}{\phi}\frac{3}{4}\frac{1}{4}\frac{3}{4}\left(\frac{6}{5}\right)^3\frac{1}{4}\frac{z^4}{a^2} \\
&= \frac{3}{4}\frac{1}{4}\frac{3}{4}\left(\frac{6}{5}\right)^3\frac{1}{4}\frac{A^2}{\phi}\frac{z^4}{a^2} \\
&= 0.0607\frac{A^2}{\phi}\frac{z^4}{a^2} < \frac{1}{16}\frac{A^2}{\phi}\frac{z^4}{a^2} = \text{ value at the first best allocation}
\end{aligned}
$$

Hence we showed that the private χ economy cannot achieve the welfare at the first best allocation.