Notes on Klein, Krusell and Rios-Rull (rough)

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Everyone understands the time consistency problem

Few understand the solution

One approach: repuation mechanisms (Chari Kehoe 1990)

Approach in KKR: look at Markov perfect equilibria. No “reputation” like state-variables. Only state variables are those determining resource feasible allocations (capital)

One motivation: reputational equilibria tend not to survive in finite horizon environments (always play static first best in last period, and reputational equilibra unravel)

Another assumption in this paper: optimal policies are smooth and differentiable. Not clear how restrictive this assumption is

1 Environment

Focus today on model with inelastic labor supply and tax on total income. Paper also considers models with elastic labor supply and with taxes on capital and labor serparately – very similar machinery

Representative agent setup

Valued governtment consumption

No debt – budget must be balanced period by period

No lump-sum taxes

Will compare allocations with commitment (Ramsey) and without (Markov).

Households maximize

$$\sum_{t=0}^{\infty} \beta^t u(c_t, g_t)$$

$$c_t + k_{t+1} = k_t + (1 - \tau_t) [w_t + (r_t - \delta) k_t]$$

where w_t and r_t are marginal products of labor and capital.

Households FOC is

$$u_c(c_t, g_t) = \beta u_c(c_{t+1}, g_{t+1}) [1 + (1 - \tau_{t+1}) r_{t+1}]$$

Resource constraint

$$c_t + k_{t+1} + g_t = f(k_t) + (1-\delta)k_t$$

GBC

$$g_t = \tau_t \left[f(k_t) - \delta k_t \right]$$

1.1 Markov equilibrium

Think of a planner as choose τ_t given initial capital k_t taking as given that future tax rates will be chosen according to the following policy rule

$$\tau_{t+j} = T(k_{t+j})$$

for all $j \geq 1$

In a Markov subgame-perfect equilibrium the policy rule T must have the property that when future policymakers follow T, the optimal choice for the policymaker at date t is to also follow the rule T. In this sense the policy maker has no incentive to make a one-shot deviation. And if the current policy maker doesn't want to deviate, neither will future policymakers (since their problem will look exactly the same)

2 Recursive formulation

Let

$$G(k,\tau) = \tau[f(k) - \delta k]$$

define the equilibrium level for g

Let

$$C(k,k',\tau) = f(k) + (1-\delta)k - k' - G(k,\tau)$$

denote the equilibrium value for current consumption given k, τ and the choice k'

Let

$$K(k,\tau)$$

denote the representative household's optimal choice for next period capital, when current capital is k, the current tax rate is τ and the household uses T to forecast future tax rates. This decision rule is implicitly defined by the household's FOC, i.e., it is the function that satisfies, for all k and τ

$$u_c(C(k,k',\tau), G(k,\tau)) = \beta u_c(C', G', T'))\left[1 + (1-T')\left(f_k(k') - \delta\right)\right]$$

where

$$\begin{aligned} k' &= K(k,\tau) \\ C' &= C(k', K(k', T(k')), T(k')) \\ G' &= G(K(k,\tau)) \\ T' &= T(K(k,\tau)) \end{aligned}$$

2.1 Planner Problem

Now consider the current planner's problem. The planner solves

$$\max_{\tau} u(C(k, K(k,\tau),\tau), G(k,\tau)) + \beta v\left(K(k,\tau)\right)$$

where the continuation value $v(k)$ corresponds to the household value achieved when $\tau = T(k)$, i.e.,

$$v(k) = u(C\left(k, K(k, T(k)\right), T(k)), G(k, T(k))) + \beta v\left(K(k, T(k))\right)$$

We solve for the optimal choice simply by taking a FOC:

$$u_c\left(C_K K_\tau + C_\tau\right) + u_G G_\tau + \beta K_\tau v'_k = 0$$

The envelope condition is

$$v_k = u_c C_k + u_g G_k + u_c C_\tau T_k + u_g G_\tau T_k$$

(where we ignore the indirect impact of k on k' in the usual way – if we include those terms they will drop out later anyway)

which gives

$$u_c\left(C_K K_\tau + C_\tau\right) + u_g G_\tau + \beta K_\tau\left[u'_c C'_k + u'_g G'_k + u'_c C'_\tau T'_k + u'_g G'_\tau T'_k\right] = 0$$

Now by the IFT, $T'_k = -\frac{K'_k}{K'_\tau}$ which we can substitute it to get

$$u_c\left(C_K K_\tau + C_\tau\right) + u_g G_\tau + \beta K_\tau\left[u'_c C'_k + u'_g G'_k - \frac{K'_k}{K'_\tau}\left(u'_c\left(C'_K + C'_\tau\right) + u'_g G'_\tau\right)\right] = 0$$

We can interpret these terms.

The first ones are easy: raising τ increases g, directly reduces c, and indirectly affects c because it changes optimal savings.

The next set of terms reflects how welfare changes because of how changing τ impacts welfare via savings: raising τ via the effect on k' directly affects c' via the budget constraint. Changing k' also mechanically changes g' through the govt budget constraint

The final set of terms reflects how welfare changes because changing τ impacts because the resulting change in k' changes τ' (the T'_k term): changing τ' changes c' directly, and also changes g'

A time consistent policy equilibrium is a set of differentiable functions T and K s.t. (1) K satisfies the HH FOC, when τ is given by $T(k)$, and (2) the planner's FOC is satisfied at $\tau = T(k)$.

3 Computation

Here we have two unknown functions $T(k)$ and $K(k)$ that must satisfy two functional equations. We could solve this using global methods.

Are things easier if we assume the time-consistent policy equilibrium converges to a steady state, and just try to characterize that steady state?

We have 2 equations. The problem is that we have 4 unknowns. The (steady state version of) the HH FOC contains two unknowns, $\bar{\tau}$ and $\bar{k}$. But the GOVT FOC brings in two more unknown constants: K_k and K_τ.

Here is the approach that KKR propose.

1. Assume K and T are constants, so $K_k = K_\tau = 0$. Solve for the steady state.

2. Assume K and ψ are affine functions s.t.

$$\begin{aligned} K(k) &= \bar{k} + \kappa_1 (k - \bar{k}) \\ T(k) &= \bar{\tau} + \tau_1 (k - \bar{k}) \end{aligned}$$

We now have 4 unknowns to solve for. So we need to add some more equations. Just differentiate both functional equations by k to get 2 more equations. Solve 4 equations in 4 unknowns to get new estimates for $\bar{k}$ and $\bar{\tau}$.

3. Keep increasing the order of the K and T equations until the steady state doesn't change much from one order to the next.

4 Interpreting the FOC

Note that $G_\tau = -C_\tau$, $C_k = f_k(k) + (1-\delta) - G_k$, $C_K = -1$

So we can write the FOC as

$$u_c \left(C_K K_\tau + C_\tau\right) + u_g G_\tau + \beta K_\tau \left[u_c' C_k' + u_g' G_k' - \frac{K_k'}{K_\tau'} \left(u_c' C_\tau' + u_g' G_\tau'\right) \right] = 0$$

$$G_\tau (u_g - u_c) + K_\tau \left[-u_c + \beta u_c' \left(f_k(k') + 1 - \delta\right)\right] + \beta K_\tau \left(G_k' - \frac{K_k'}{K_\tau'} G_\tau' \right) \left(u_g' - u_c'\right) = 0$$

So the optimal policy trades off wedges. In the first best, we would have $u_g = u_c$ and $u_c = \beta u_c' \left(f_k(k') + 1 - \delta\right)$. But in the Markov equilibrium, neither of these will be quite zero.

5 Quantification

With capital taxes only, find higher capital taxes in steady state compared to Ramsey solution. But tax not set to the level that equates marginal utility of public and private consumption, even though the tax only applies to current capital, which is already in place. The logic is that a lower capital tax rate implies more saving, and with more capital in place tomorrow, tomorrow's planner will be less tempted to impose a very high capital tax rate. Or perhaps better logic: the planner today expects future planners to choose high and distortionary taxes, which will depress future output and consumption. By choosing a relatively low capital tax today, the planner ensures more saving, which partially offsets the impact of future distortions.